Azonosító jel: | | | | | | | | | | | | | | | |

ÉRETTSÉGI VIZSGA • 2024. október 15.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2024. október 15. 8:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

OKTATÁSI HIVATAL

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. **a)** Let a and b be two positive real numbers. Determine the truth value of the following statement (true or false). Explain your answer.

"If $a > 6$ and $b > 8$ then the arithmetic mean of a and b is greater than 7."

b) State the converse of the above statement and determine the truth value of the converse. Explain your answer.

c) Determine the value of the positive real number x such that the harmonic mean of 7 and x will be 10.

d) It is known that the truth value of the statement $(A \wedge \neg B) \wedge (A \vee \neg C)$ is true.
What can be said about the truth values of statements A, B and C? Mark the appropriate cell of the table below with X. (Here you do not need to justify your answers.)

	certainly true	certainly false	may not be determined
A			
B			
C			

a)	2 points	
b)	3 points	
c)	4 points	
d)	3 points	
T:	12 points	

2. When a cup of hot liquid is placed into a room, the temperature change of the liquid follows Newton's Law of Cooling and can be modelled approximately by the formula $T(t) = H + (T(0) - H) \cdot 2^{ct}$.

In this formula $T(t)$ is the temperature of the liquid t minutes after the beginning of the experiment, H is the (constant) temperature of the room, $T(0)$ is the initial temperature of the liquid ($T(0) > H$), while c is a constant that is characteristic of the cooling liquid. (Temperatures here are measured in °C.)

A cup of 75 °C coffee is placed into a 25 °C room. The value of c in case of coffee is –0.209.

a) Calculate the temperature of the coffee 15 minutes later.

b) How long after the beginning of the experiment will the temperature of the coffee drop to 25.5 °C?

A cup of 85 °C coffee is placed into a room of unknown temperature. In 10 minutes, the coffee will cool to 40 °C.

c) Determine the temperature of the room.

a)	4 points	
b)	4 points	
c)	4 points	
T:	12 points	

3. **a)** Solve the following equation over the set of real numbers.

$$\left|2\sin^2 x + 7\sin x + 1\right| = 5$$

b) Determine the area of the bounded region enclosed by the graph of the function $f\colon \mathbf{R} \to \mathbf{R},\ x \mapsto \sin x$, the lines $x = \dfrac{\pi}{6}$ and $x = \dfrac{5\pi}{6}$, and the x axis.

a)	8 points	
b)	4 points	
T:	12 points	

4. Consider all triangles where the lengths of the three sides a, b and c (measured in centimetres) are such that $b = a + 4$ and $c = a + 8$.

a) The measure of the greatest angle in one such triangle is 120°. Determine the length of the sides of this triangle.

b) The length of the longest side in one such triangle is 24 cm. Determine the area of this triangle.

c) Prove that all such triangles will have a perimeter longer than 24 cm.

a)	6 points	
b)	5 points	
c)	4 points	
T:	15 points	

II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

5. A fair gambling dice is thrown six times. The numbers shown in monotone increasing order are 1, 2, 2, 3, 3, 3.

a) Determine the mean and the standard deviation of the numbers thrown.

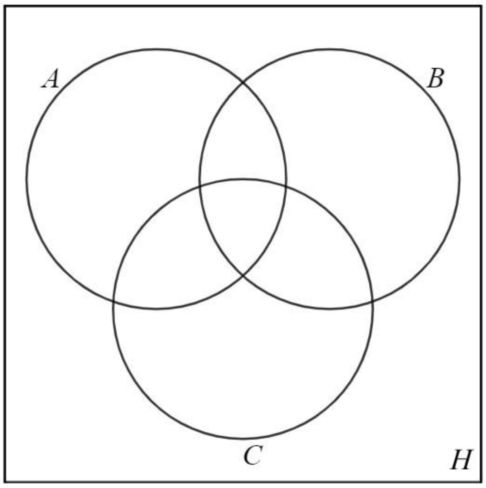
b) In how many different orders may a single 1, two 2-s and three 3-s be obtained?

A fair gambling dice is thrown twice.

c) Determine the probability that the product of the two numbers shown will be divisible by 2, but not divisible by 4.

One blue and one green dice is thrown, the outcome is an ordered pair of numbers. Let (k, z) be the outcome when the number shown on the blue dice is k and the number shown on the green one is z.
Let the fundamental set H consist of all possible pairs of numbers (k, z) that are outcomes of this experiment. Define the subsets A, B and C as follows:
$A = \{(k, z) \mid \text{the sum } k + z \text{ is a prime}\}$
$B = \{(k, z) \mid \text{the product } k \cdot z \text{ is a prime}\}$
$C = \{(k, z) \mid k = z\}$

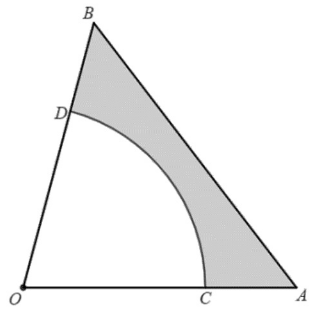
d) Shade the region of H on the Venn-diagram shown here that is an empty subset. Write an appropriate pair of numbers into every other region of the Venn-diagram. You do not need to justify your answers here.



a)	3 points	
b)	3 points	
c)	5 points	
d)	5 points	
T:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

6. Sides OA and OB in the isosceles triangle OAB are both 12 cm long, the measure of the angle AOB is 75°. Connect point C on side OA and point D on side OB with an arc, the centre of which is point O and the radius of which is 8 cm (as shown in the diagram).

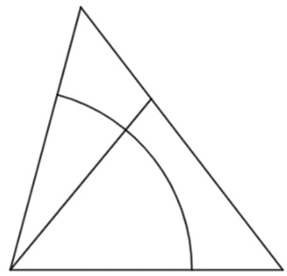


a) Determine the area and the perimeter of the grey shaded region of the diagram.

Rotate triangle OAB around the line of side OA.

b) Determine the volume of the solid of revolution obtained.

The four regions of the diagram are coloured in red, blue and green, such that each region is of a single colour.

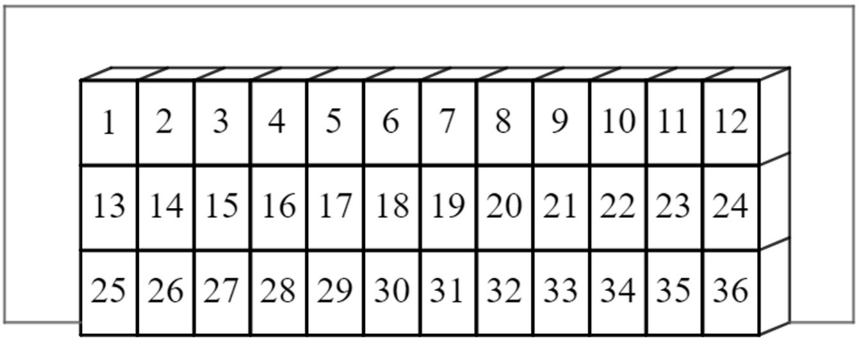


c) How many different ways are there to colour the four regions if adjacent regions must not be of the same colour?
(Two regions are adjacent if they share a common borderline. It is not required to use all three colours for the colouring.)

a)	8 points	
b)	4 points	
c)	4 points	
T:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

7. There are 36 cabinets for the students in Alíz's classroom, in three rows, 12 cabinets in each row, numbered 1-36. There are 33 students in the class. At the beginning of the school year each student is randomly assigned a cabinet, the three remaining cabinets will be left empty.



a) Let event *A* be that the three empty cabinets will be in the same row and let event *B* be that the three empty cabinets will be in three different rows. Which event has a higher probability?

The shape of each cabinet is a cuboid. The inner dimensions of a cabinet are the following: 20 cm wide, 35 cm tall and 30 cm deep.

b) Determine the length of the longest straight rod that could be placed inside these cabinets. (Ignore the thickness of the rod.)

Alíz, Boglárka, Csenge and Dorka accidentally swapped their cabinet keys and now they are distributing the four keys randomly among the four of them.

c) Determine the probability that at least two of the four girls would get their own keys back.

a)	6 points	
b)	3 points	
c)	7 points	
T:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

8. The combined total mass of 1000 seeds of a particular species of plant is called the **1,000 kernel (1,000 K) weight**. The 1,000 K weight of a certain species of oats is 35 grams.

a) What is the approximate number of seeds in one ton of this type of oats? Give your answer in scientific form.

While on a summer job, Jancsi is sharpening oat seeds*. He knows that k kilograms of oats, when placed into the sharpening machine, will be processed in $\frac{k^2}{40}+90$ minutes. Jancsi needs to process 1000 kg of oats with this machine. He decides to do the job dividing the oats into equal lots.

b) How many hours will it take to process the 1000 kg of oats if it is divided into 8 equal lots?

c) Divide the 1000 kg of oats into n equal lots ($n \in \mathbf{Z}^+$). Determine the value of n such that the time it takes to process all l000 kg of it will take the least possible amount of time. What is this minimum amount of time?

a)	3 points	
b)	3 points	
c)	10 points	
T:	16 points	

*This is an untranslatable pun referring to a Hungarian proverb.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

9. The n^{th} term of a geometric sequence is $a_n = 2 \cdot \left(-\frac{1}{2}\right)^n$ ($n \in \mathbf{Z}^+$).

a) Determine the smallest possible value of n where $|a_n| < 10^{-7}$ is true.

b) Determine the sum of the first 10 terms of this geometric sequence.
Give your answer in $-\frac{k}{m}$ form, where k and m are relatively primes (co-primes).

The n^{th} term of the sequence $\{b_n\}$ is $b_n = 2 \cdot \left(-\frac{1}{2}\right)^n + 2$ ($n \in \mathbf{Z}^+$).

c) Prove that (for all positive integer value of n) $2b_{n+2} - b_{n+1} - b_n = 0$.

a)	6 points	
b)	4 points	
c)	6 points	
T:	16 points	

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<tr><td></td><td rowspan="2">Number of problem</td><td colspan="4">score</td></tr>
<tr><td>maximum</td><td>awarded</td><td>maximum</td><td>awarded</td></tr>
<tr><td rowspan="4">Part I</td><td>1.</td><td>12</td><td></td><td rowspan="4">51</td><td rowspan="4"></td></tr>
<tr><td>2.</td><td>12</td><td></td></tr>
<tr><td>3.</td><td>12</td><td></td></tr>
<tr><td>4.</td><td>15</td><td></td></tr>
<tr><td rowspan="5">Part II</td><td></td><td>16</td><td></td><td rowspan="4">64</td><td rowspan="4"></td></tr>
<tr><td></td><td>16</td><td></td></tr>
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date

examiner

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<tr><td></td><td>elért</td><td>programba beírt</td></tr>
<tr><td>I. rész</td><td></td><td></td></tr>
<tr><td>II. rész</td><td></td><td></td></tr>
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javító tanár

jegyző