ÉRETTSÉGI VIZSGA • 2024. október 15.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without that remark as well.

6. Deduct points for **missing units** only if the missing unit is part of the answer or a unit exchange (without parentheses).
7. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1. a)		
The statement is true.	1 point	
Correct reasoning, for example: $\frac{a+b}{2} > \frac{6+8}{2} = 7$.	1 point	
Total:	**2 points**	

1. b)		
The converse: “If the arithmetic mean of a and b is greater than 7 then $a > 6$ and $b > 8$.”	1 point	
The converse is false.	1 point	
Appropriate counter-example (e.g. $a = 5$, $b = 11$).	1 point	
Total:	**3 points**	

1. c)		
(The harmonic mean of 7 and x is $\frac{2}{\frac{1}{7}+\frac{1}{x}}$, so) the equation $\frac{2}{\frac{1}{7}+\frac{1}{x}} = 10$ is to be solved (where $x > 0$).	1 point	*Award this point if the candidate uses the form $\frac{2ab}{a+b}$ for the harmonic mean..*
$\frac{14x}{x+7} = 10$	1 point	
$14x = 10x + 70$	1 point	
$x = 17.5$.	1 point	
Total:	**4 points**	

1. d)

	certainly true	certainly false	may not be determined
A	X		
B		X	
C			X

1-1 point

Total: **3 points**

2. a)		
$T(0) = 75$ °C, $H = 25$ °C, $t = 15$ minutes, $c = –0.209$	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
$T(15) = 25 + (75 - 25) \cdot 2^{-0.209 \cdot 15} =$	1 point	
$= 25 + 50 \cdot 2^{-3.135} \approx$	1 point	
≈ 30.7 °C is what the temperature of the coffee will be 15 minutes later.	1 point	
Total:	**4 points**	

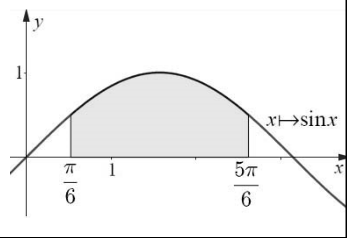
2. b)		
$25.5 = 25 + 50\cdot 2^{-0.209\cdot t}$	1 point	
$0.01 = 2^{-0.209\cdot t}$	1 point	
$-0.209t = \log_2 0.01 \approx -6.644$	1 point	$-0.209t = \frac{\log 0.01}{\log 2}$
$t \approx 31.8$, so it will take approximately 32 minutes until the temperature of the coffee drops to 25.5 °C.	1 point	
Total:	**4 points**	

2. c)		
$40 = H + (85 - H)\cdot 2^{-0.209\cdot 10}$	1 point	
$40 = H + (85 - H)\cdot 0.235$	1 point	
$40 = 0.765H + 19.975$	1 point	
The temperature of the room is $H \approx 26.2$ °C.	1 point	
Total:	**4 points**	

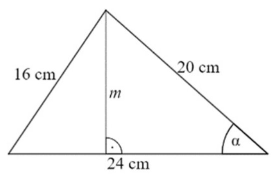
Note: Deduce a total of 1 point for the whole Question 2 if the candidate gives their answers without a unit more than once.

3. a)		
Case I: $2\sin^2 x + 7\sin x + 1 = 5$, i.e. $2\sin^2 x + 7\sin x - 4 = 0$.	1 point	
It is a quadratic equation for ($\sin x$), the roots are 0.5 and –4.	1 point	
Case II: $2\sin^2 x + 7\sin x + 1 = -5$, i.e. $2\sin^2 x + 7\sin x + 6 = 0$.	1 point	
It is a quadratic equation for ($\sin x$), the roots are –1.5 and –2.	1 point	
(As $-1 \le \sin x \le 1$) the only possible root of the four is $\sin x = 0.5$,	1 point	
when $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{5\pi}{6} + 2k\pi$, $k \in \mathbf{Z}$.	2 points	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**8 points**	

Notes:
1. Deduce 1 point if the candidate gives their correct answer in degrees.
2. Deduce 1 point if the candidate gives their correct answer without the periods.

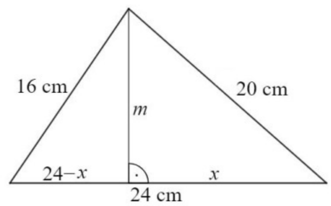
3. b)		
(The function is positive over the interval $\left[\frac{\pi}{6};\frac{5\pi}{6}\right]$ so) the area is $A=\int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\sin x\,dx=$	1 point	
$=\left[-\cos x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}=$	1 point	
$=-\left(-\frac{\sqrt{3}}{2}\right)-\left(-\frac{\sqrt{3}}{2}\right)=$	1 point	
$=\sqrt{3}$ (≈ 1.73).	1 point	
Total:	**4 points**	

4. a)		
The 120° angle is the greatest angle in the triangle thus being opposite to the longest side (c).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
Apply the Cosine Rule to side c: $(a+8)^2=a^2+(a+4)^2-2a(a+4)\cdot\cos 120°$.	1 point	
Rearranged: $0=2a^2-4a-48$.	2 points	
The positive solution of the equation is $a=6$ (the negative solution is –4, which is impossible).	1 point	
The three sides of the triangle are 6 (cm), 10 (cm) and 14 (cm) (which also satisfies the triangle-inequality).	1 point	
Total:	**6 points**	

4. b) Solution 1		
The other two sides are 16 cm and 20 cm.	1 point	
Let α be the angle opposite the 16 cm side. Apply the Cosine Rule: $\cos\alpha=\frac{24^2+20^2-16^2}{2\cdot 24\cdot 20}=0.75$.	1 point	
$\alpha\approx 41.4°$.	1 point	
The area of the triangle: $A=\frac{24\cdot 20\cdot\sin 41.4°}{2}\approx$	1 point	*The height that belongs to the* 24 cm *side is:* $m=20\cdot\sin 41.4°\approx 13.2$ cm.
≈ 158.7 cm^2.	1 point	$A=\frac{24\cdot 13.2}{2}\approx 158.4$ cm^2
Total:	**5 points**	

Note: The other two angles of the triangle are 55.8° and 82.8°. The other two heights are 15.87 cm *and* 19.84 cm.

4. b) Solution 2		
The other two sides are 16 cm and 20 cm.	1 point	
The semi-perimeter of the triangle is: $s = \left(\frac{24+20+16}{2} = \right) 30$ cm.	1 point	
Use Heron's Formula: $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{30 \cdot 14 \cdot 10 \cdot 6} =$	2 points	
$= 60\sqrt{7} \approx 158.7$ cm^2.	1 point	
Total:	**5 points**	

4. b) Solution 3		
The other two sides are 16 cm and 20 cm.	1 point	
Let m be the height that belongs to the 24 cm side. (This height is inside the triangle.) This height divides the base into two sections, x cm and $24 - x$ cm long (as shown in the diagram). Express m^2 from the Pythagorean Theorem applied for both triangles: $(m^2 =)\ 20^2 - x^2 = 16^2 - (24-x)^2$.	1 point	
$400 - x^2 = 256 - 576 + 48x - x^2$, then $x = 15$ cm.	1 point	
$m = \sqrt{20^2 - x^2} = \sqrt{175} \approx 13.2$ cm.	1 point	
$A = \frac{24 \cdot 13.2}{2} \approx 158.4$ cm^2.	1 point	
Total:	**5 points**	

4. c)		
As c is the longest side, a triangle is only possible if $a + b > c$ (according to the triangle-inequality).	1 point	
$a + (a + 4) > a + 8$,	1 point	
$a > 4$.	1 point	
So, the perimeter of the triangle must be greater than $(4 + 8 + 12 =)$ 24 cm, indeed.	1 point	
Total:	**4 points**	

II.

5. a)		
The mean is $\frac{1+2+2+3+3+3}{6}=\frac{7}{3}$ (≈ 2.33).	1 point	*Award these points if the candidate correctly determines the mean and the standard deviation using a calculator.*
The standard deviation is $\sqrt{\frac{\left(\frac{4}{3}\right)^2+2\cdot\left(\frac{1}{3}\right)^2+3\cdot\left(\frac{2}{3}\right)^2}{6}}=\frac{\sqrt{5}}{3}\approx 0.745.$	2 points	
Total:	**3 points**	

5. b)		
The number 1 may be on any of six position in a certain order of throws.	1 point	*Counting by permutation with repeat:* $\frac{6!}{2!\cdot 3!}=$
There are $\binom{5}{2}=10$ different ways to select the position of the two 2-s out of the remaining five throws. (The 3-s will take the remaining three positions.)	1 point	
There are a total of $6\cdot 10 = 60$ possible different orders.	1 point	$= 60.$
Total:	**3 points**	

5. c) Solution 1		
There are 36 different outcomes for two throws (total number of cases).	1 point	
Favourable are the outcomes where one number is even, but not divisible by 4 (a 2 or a 6) and the other number is odd (1, 3 or 5).	1 point	
Any pair of appropriate numbers may be obtained in two different orders, the number of favourable cases is, therefore, $2\cdot 2\cdot 3 = 12$.	2 points	
The probability is $\frac{12}{36}=\frac{1}{3}$ (≈ 0.333).	1 point	
Total:	**5 points**	

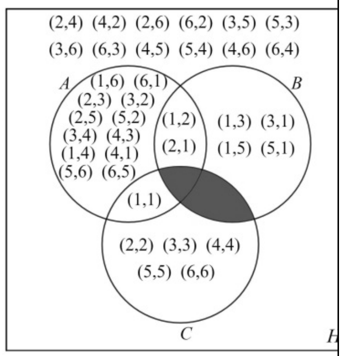
5. c) Solution 2		
Use complement events. Unfavourable are the outcomes when the product is either not divisible by 2 or is divisible by 4.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
Case I. The product is not divisible by 2 if two odd numbers are thrown. The probability of this is $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$.	1 point	
Case II/1. The product is divisible by 4 if one of the numbers thrown is 4 and the other is an odd number. The probability of this is $2\cdot\frac{1}{6}\cdot\frac{1}{2}=\frac{1}{6}$.	1 point*	
Case II/2. The product is also divisible by 4 if both numbers thrown are even. The probability of this is $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$.	1 point*	
The final probability: $1-\frac{1}{4}-\frac{1}{6}-\frac{1}{4}=\frac{1}{3}$ (≈ 0.333).	1 point*	
Total:	**5 points**	

*Notes: 1. The 3 points marked * may also be given for the following reasoning:*

Case II/1. The product is divisible by 4 if one of the numbers shown is 4 (and the other is any number). The probability of this is $\frac{6+6-1}{36}=\frac{11}{36}$.	1 point	
Case II/2. The product is also divisible by 4 if both numbers thrown are even, but not 4. The probability of this is $\frac{2}{6}\cdot\frac{2}{6}=\frac{4}{36}$.	1 point	
The final probability is $1-\frac{9}{36}-\frac{11}{36}-\frac{4}{36}=\frac{12}{36}$.	1 point	

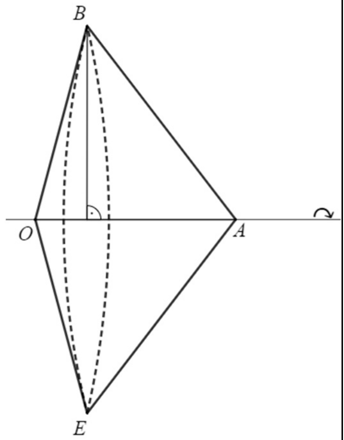
2. Award maximum score if the candidate gives the correct answer using a logically ordered list of all possibilities (as, for example, is shown in the table below).

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

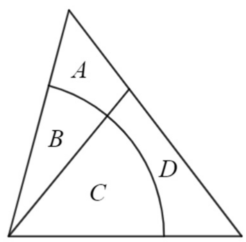
<table>
<tr><td colspan="3">5. d)</td></tr>
<tr><td>A correct set diagram.</td><td>5 points</td><td>
</td></tr>
<tr><td>Total:</td><td>5 points</td><td></td></tr>
</table>

Note: Award 5 points for 8 correctly filled regions. Award 4 points for 7 correctly filled regions. Award 3 points for 5 or 6 correctly filled regions. Award 2 points for 3 or 4 correctly filled regions. Award 1 point for 1 or 2 correctly filled regions.
A non-empty region is filled correctly if there is at least one appropriate pair of numbers in it and no inappropriate pair. An empty region is correctly filled if it is shaded.

6. a)		
The area of triangle OAB is $\frac{12^2 \cdot \sin 75°}{2} \approx 69.5\ \text{cm}^2$.	1 point	
The area of the sector OCD is $\frac{75°}{360°} \cdot 8^2 \cdot \pi \approx 41.9\ \text{cm}^2$.	1 point	
The area of the grey region is $(69.5 - 41.9 =)\ 27.6\ \text{cm}^2$.	1 point	
Apply the Cosine Rule in triangle OAB: $AB = \sqrt{12^2 + 12^2 - 2 \cdot 12 \cdot 12 \cdot \cos 75°} \approx$	1 point	
≈ 14.6 cm.	1 point	
The length of the arc CD is $\frac{75°}{360°} \cdot 2 \cdot 8 \cdot \pi \approx 10.5$ cm.	1 point	$\frac{2A_{OCD}}{r} = \frac{2 \cdot 41.9}{8} \approx 10.5$ cm
$CA = DB = 12 - 8 = 4$ cm	1 point	
The perimeter of the grey region is $(14.6 + 4 + 10.5 + 4 =)\ 33.1$ cm.	1 point	
Total:	**8 points**	

6. b)		
The height from vertex B in triangle OAB is $12 \cdot \sin 75° \approx 11.6$ cm.	1 point	
The solid of revolution obtained consists of two straight cones. The radius of the base is 11.6 cm for both cones, the sum of their heights is OA = 12 cm.	2 points	*One height is* $12 \cdot \cos 75° \approx 3.1$ cm, *the other is* 8.9 cm.
The volume of the solid of revolution is $\frac{11.6^2 \cdot \pi \cdot 12}{3}$ ≈ 1691 cm^3.	1 point	437 cm^3 + 1254 cm^3
Total:	**4 points**	

Note: Deduce a total of 1 point for the whole Question 6 if the candidate gives their answers without a unit more than once.

6. c) Solution 1		
Use the letters from the diagram on the right. Two adjacent regions, for example A and B can be coloured in 3·2 = 6 different ways.	1 point	
Now, if region C is of the same colour as A then region D can be coloured in 2 different colours.	1 point	
If, however, C is coloured in the third colour, then there is only one colour left for D (in this case B and D are of the same colour).	1 point	
This gives a total 6·(2 + 1) = 18 different options.	1 point	
Total:	**4 points**	

6. c) Solution 2		
When using only two colours, there are 3 different ways to select these. Each pair of colours could be used in two different ways to colour the diagram. That is $(3\cdot 2 =)$ 6 options.	1 point	
When using three colours, there are 3 different ways to pick the colour that is used for two regions, 2 ways to select the two (opposite) regions coloured with this colour, and 2 ways to colour the remaining two regions. That is $(3\cdot 2\cdot 2 =)$ 12 options.	2 points	
There are a total of $(6 + 12 =)$ 18 possible options.	1 point	
Total:	**4 points**	

Note: Award maximum score if the candidate gives the correct answer using a logically ordered list of all possibilities.

7. a) Solution 1		
(The order of selection is not considered, only the three remaining cabinets are. Any three cabinets are left empty at equal probability.) There are $\binom{36}{3}$ $(= 7140)$ ways to select three cabinets out of 36 (total number of cases).	1 point	*(The cabinets assigned to students are considered.)* $\binom{36}{33}$ $(= 7140)$
There are $\binom{12}{3}$ $(= 220)$ ways to select three cabinets from a particular row. As there are three rows, the number of favourable cases is $3\cdot\binom{12}{3}$ $(= 660)$.	1 point	$3\cdot\binom{12}{9}\binom{12}{12}\binom{12}{12}$ $(= 660)$
The probability of event A is then $\dfrac{3\cdot\binom{12}{3}}{\binom{36}{3}} \approx 0.092$.	1 point	
There are $12\cdot 12\cdot 12$ $(=1728)$ different ways to select one cabinet from each of the three rows.	1 point	$\binom{12}{11}^3$ $(= 1728)$
The probability of event B is then $\dfrac{12\cdot 12\cdot 12}{\binom{36}{3}} \approx 0.242$.	1 point	
The probability of event B is higher.	1 point	
Total:	**6 points**	

7. a) **Solution 2**		
(The order of selection is also considered. Assume, that the empty cabinets will be selected first.) The first cabinet selected could be any of them (in both cases).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
In the first case, there are 11 "good" cabinets out of the remaining 35 for the second selection and 10 out of the remaining 34 for the third (and the selections are independent), and so the probability of event A is $\frac{11}{35}\cdot\frac{10}{34}\approx 0.092$.	2 points	
In the second case, there are 24 "good" cabinets out of the remaining 35 for the second selection and 12 out of the remaining 34 for the third, and so the probability of event B is $\frac{24}{35}\cdot\frac{12}{34}\approx 0.242$.	2 points	
The probability of event B is higher.	1 point	
Total:	**6 points**	

7. a) **Solution 3**		
(The order of selection is also considered. Assume, that the empty cabinets will be selected first.) There are $36\cdot35\cdot34$ (= 42 840) different ways to select three cabinets out of 36 (total number of cases).	1 point	
In case of event A, any cabinet can be selected first, and then there are 11 and 10 options (in the row where the first cabinet is) for the second and third selections. The number of favourable cases is $36\cdot11\cdot10$ (= 3960).	2 points	
In case of event B, any cabinet can be selected first, and then there are 24 and 12 options for the second and third selections. The number of favourable cases is $36\cdot24\cdot12$ (= 10 368).	2 points	
As the second case has more favourable outcomes (while the total is the same in both), the probability of event B is higher.	1 point	
Total:	**6 points**	

7. b)		
The longest rod may be placed alongside the solid diagonal of the cuboid.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The length of the solid diagonal is $\sqrt{20^2+35^2+30^2}\approx$	1 point	
$\approx$ 50 cm, this is the length of the longest straight rod that could be placed inside these cabinets.	1 point	
Total:	**3 points**	

7. c) **Solution 1**		
There are 4! (= 24) different ways to distribute the keys (total number of cases).	1 point	
Favourable cases: I. All four of them may get their own keys back. This is 1 possible case.	1 point	*ABCD*
It is impossible that exactly three of them would get their own keys back (as the fourth girl must also get hers back then).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
II. If exactly two of the girls get their own keys back than the other two girls get theirs switched around.	1 point	*ABDC, ADCB, ACBD, DBCA, CBAD, BACD (here each key is denoted by the initial of their respective owners and the keys are distributed in the following order: Alíz, Boglárka, Csenge, Dorka).*
There are $\binom{4}{2} = 6$ ways to select the two girls who will receive their own keys (and then it is obvious how the other two girls receive their keys).	1 point	
The number of favourable cases is then (1 + 6 =) 7.	1 point	
The probability is $\frac{7}{24} \approx 0.292$.	1 point	
Total:	**7 points**	

7. c) **Solution 2**		
The probability that, one by one, they each receive their own keys is $\frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot 1=\frac{1}{24}$.	2 points	
It is impossible that exactly three of them would get their own keys back (as the fourth girl must also get hers back then).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The probability that, for example, Alíz and Boglárka are receiving their own keys but Csenge and Dorka are not, is $\frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot 1=\frac{1}{24}$.	2 points	
There are $\binom{4}{2} = 6$ different ways to select the two girls who will receive their own keys, so the probability that exactly two girls will receive their own keys is $6\cdot\frac{1}{24}=\frac{6}{24}$.	1 point	
The final probability is $\left(\frac{1}{24}+\frac{6}{24}=\right)\frac{7}{24} \approx 0.292$.	1 point	
Total:	**7 points**	

8. a)		
1 ton (= 1000 kg) = 1 000 000 grams	1 point	
(There are 1000 oat seeds in every 35 grams.) One million grams of seeds is $\frac{1\,000\,000}{35}\cdot 1000 \approx$ ≈ 28.57 million,	1 point	
So, there are about $2.857\cdot 10^7$ seeds in a ton.	1 point	
Total:	**3 points**	

8. b)		
When 1000 kg of oats is divided into 8 lots, each lot is 125 kg.	1 point	
It then takes $8\cdot\left(\frac{125^2}{40}+90\right)=3845$ minutes for the machine to process it,	1 point	
which is equivalent to $\left(\frac{3845}{60}\approx\right)$ 64 hours.	1 point	
Total:	**3 points**	

8. c)		
When the 1000 kg of oats is divided into n lots, the mass of each lot will be $\frac{1000}{n}$ kg.	1 point	
This will take $n\cdot\left(\frac{1}{40}\cdot\frac{1000^2}{n^2}+90\right)=\frac{25\,000}{n}+90n$ minutes for the machine to process all the oats.	2 points	
Examine the function $f\colon \mathbf{R}^+ \to \mathbf{R}$, $f(x)=\frac{25\,000}{x}+90x$. This function will have a minimum, where its first derivative is zero.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
$f'(x)=-\frac{25\,000}{x^2}+90=0$	1 point	
$x=\sqrt{\frac{25\,000}{90}}=\frac{50}{3}\approx 16.67$ (as $x>0$).	1 point	
If $x > 16.67$ then $f'(x)>0$, and if $0<x<16.67$, then $f'(x)<0$, and so it really is a minimum of the function f.	1 point	$f''(x)=\frac{50\,000}{x^3}>0$

(As $n \in \mathbf{Z}^+$, the minimal time will be obtained at either $n = 16$ or $n = 17$.) $f(16) = 3002.5$ and $f(17) = 3000.6$,	1 point	
so the time to process 1000 kg of oats is minimal when it is divided into 17 lots.	1 point	
The minimal time is then about $\left(\frac{3000}{60}=\right)$ 50 hours.	1 point	*The answer is also acceptable in minutes.*
Total:	**10 points**	

9. a)		
The inequality $\left\|2\cdot\left(-\frac{1}{2}\right)^n\right\| < 10^{-7}$ is to be solved,	1 point	
i.e. $\frac{1}{2^{n-1}} < 10^{-7} \Leftrightarrow 10^7 < 2^{n-1}$.	1 point	
As the base 2 logarithm function is (strictly) monotone increasing,	1 point	
$\log_2 10^7 < n-1$,	1 point	
$n > \log_2 10^7 + 1 \approx 24.3$.	1 point	
The value of n is 25.	1 point	
Total:	**6 points**	

Note: Award maximum score if the candidate gives the correct answer by calculating the first 25 terms of the sequence using reasonable and correct rounding.

9. b)		
The first term of the sequence is $a_1 = 2\cdot\left(-\frac{1}{2}\right) = -1$,	1 point	
the common ratio is $q = -\frac{1}{2}$.	1 point	
The sum of the first 10 terms: $S_{10} = a_1 \cdot \frac{q^{10}-1}{q-1} =$ $= (-1)\cdot\frac{\left(-\frac{1}{2}\right)^{10}-1}{-\frac{1}{2}-1} = -\frac{-\frac{1023}{1024}}{-\frac{3}{2}} = -\frac{2046}{3072}$,	1 point	
in the required form it is $-\frac{341}{512}$.	1 point	
Total:	**4 points**	

9. c) **Solution 1**		
Using the explicit form: $2b_{n+2}-b_{n+1}-b_n=$ $=2\cdot\left(2\cdot\left(-\frac{1}{2}\right)^{n+2}+2\right)-\left(2\cdot\left(-\frac{1}{2}\right)^{n+1}+2\right)-$ $-\left(2\cdot\left(-\frac{1}{2}\right)^{n}+2\right)=$	2 points	
$=4\cdot\left(-\frac{1}{2}\right)^{n}\cdot\frac{1}{4}+4-2\cdot\left(-\frac{1}{2}\right)^{n}\cdot\left(-\frac{1}{2}\right)-2-$ $-2\cdot\left(-\frac{1}{2}\right)^{n}-2=$	2 points	
$=\left(-\frac{1}{2}\right)^{n}\cdot\left(4\cdot\frac{1}{4}-2\cdot\left(-\frac{1}{2}\right)-2\right)+4-2-2=$	1 point	
$=\left(-\frac{1}{2}\right)^{n}\cdot(1+1-2)+0=0$ which is correct, indeed, for all values of n.	1 point	
Total:	**6 points**	

9. c) **Solution 2**		
The "constant part" of $2b_{n+2}$ is 4, while the constant part of b_{n+1} and b_n is 2, and so the sum of the constants in the expression $2b_{n+2}-b_{n+1}-b_n$ is $4-2-2=0$.	2 points	$b_n=a_n+2$, *so* $2b_{n+2}-b_{n+1}-b_n=$ $=2(a_{n+2}+2)-(a_{n+1}+2)-$ $-(a_n+2)=$
The "non-constant" part is the same as the geometric sequence $\{a_n\}$ in part a) of this question.	1 point	$=2a_{n+2}-a_{n+1}-a_n$
$2a_{n+2}-a_{n+1}-a_n=2a_n\cdot\left(-\frac{1}{2}\right)^{2}-a_n\cdot\left(-\frac{1}{2}\right)-a_n=$	1 point	
$=a_n\cdot\left(\frac{1}{2}+\frac{1}{2}-1\right)=0$	1 point	
The sum of the non-constant parts is also 0, therefore $2b_{n+2}-b_{n+1}-b_n=0$ will be true for all values of n.	1 point	
Total:	**6 points**	

9. c) Solution 3		
Express both b_{n+1} and b_{n+2} with b_n. $b_{n+1}=(b_n-2)\cdot\left(-\frac{1}{2}\right)+2=-\frac{1}{2}b_n+3$	2 points	
$b_{n+2}=(b_n-2)\cdot\frac{1}{4}+2=\frac{1}{4}b_n+\frac{3}{2}$	2 points	
Now $2b_{n+2}-b_{n+1}-b_n=\frac{1}{2}b_n+3+\frac{1}{2}b_n-3-b_n=$	1 point	
$=b_n\cdot\left(\frac{1}{2}+\frac{1}{2}-1\right)=0$ will be true for all values of n.	1 point	
Total:	**6 points**	

9. c) Solution 4		
Rearrange the inequality: $b_{n+2}=\frac{b_{n+1}+b_n}{2}$. (Now it is to be proven that, starting from the 3rd term, each term is the arithmetic mean of the previous two terms.)	1 point	
$2\cdot\left(-\frac{1}{2}\right)^{n+2}+2=\frac{2\cdot\left(-\frac{1}{2}\right)^{n+1}+2+2\cdot\left(-\frac{1}{2}\right)^{n}+2}{2}$	1 point	
Applying the division on the right side: $2\cdot\left(-\frac{1}{2}\right)^{n+2}+2=\left(-\frac{1}{2}\right)^{n+1}+1+\left(-\frac{1}{2}\right)^{n}+1$.	1 point	
$2\cdot\frac{1}{4}\cdot\left(-\frac{1}{2}\right)^{n}=\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)^{n}+\left(-\frac{1}{2}\right)^{n}$	1 point	
dividing by $\left(-\frac{1}{2}\right)^{n}$: $\frac{1}{2}=\left(-\frac{1}{2}\right)+1$, which is an identity.	1 point	
As all steps of the solution were equivalent, the original statement is now proven.	1 point	
Total:	**6 points**	