Azonosító jel:

ÉRETTSÉGI VIZSGA • 2025. október 14.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ
ÍRÁSBELI VIZSGA

2025. október 14. 8:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

OKTATÁSI HIVATAL

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. When solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used in answering the questions. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **is acceptable without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in *Négyjegyű függvénytáblázat* (Formula Booklet) (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. When solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it out, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. **a)** Solve the following equation over the set of real numbers.

$9^{x+1} + 78 \cdot 3^{x-1} - 3 = 0$

b) The second term of the geometric sequence $\{b_n\}$ is 48, the fifth term is 162. Determine the value of n such that $b_n > 10\,000\,000$.

a)	6 points	
b)	7 points	
T.:	13 points	

2. There were two different options to buy tickets for a school ball. Early bird tickets cost 2500 Ft each, while on-site tickets sold for 3000 Ft apiece. A total of 917 tickets were sold, generating a total revenue of 2 380 000 Ft for the organisers.

a) How many on-site tickets and how many early bird tickets were sold?

Attendants of the ball may try their luck with a game: 40 toy boats are floating in a tub, 5 of which are winners. Petra pulls out two boats from the tub, without replacement. (Whether a boat is a winner or not is only revealed after it has been pulled out from the tub.)

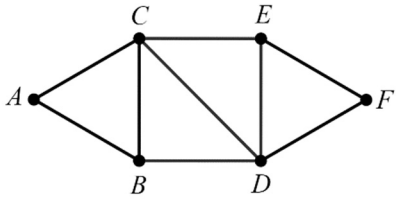
b) Calculate the probability that exactly one out of the two boats is a winner.

The ball also featured a game show with three contestants having qualified to the final round: Anna, Bálint and Csilla.

c) In how many different orders can the three contestants finish the final round if both two-way ties and three-way ties may occur?

a)	6 points	
b)	3 points	
c)	5 points	
T.:	14 points	

3. The diagram on the right shows a graph with 6 vertices and 9 edges.



a) How many different routes are possible from vertex A to vertex F if any vertex and any edge may only be travelled through at most once?

In the statement below a, b and c refer to positive real numbers.
"If there exists a triangle whose sides (measured in centimetres) are a, b and c long, then $a^2 + 2ab + b^2 - c^2 > 0$ is true."

b) Prove that the above statement is true.

c) State the converse of the statement and determine the truth value (true or false) of the converse. Explain your answer.

a)	5 points	
b)	4 points	
c)	3 points	
T.:	12 points	

4. Consider points $A(-12; 21)$ and $B(6; -3)$ given in the right-angled coordinate system.

a) Give the equation of line f, the perpendicular bisector of the segment determined by the above points and calculate the angle (in degrees) between the line f and the y-axis.

The radius of a circle is 26 units. This circle passes through the point $P(24; 6)$, and its centre is on the y-axis.

b) Determine the equation of this circle.

a)	6 points	
b)	6 points	
T.:	12 points	

II.

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

5. A die was rolled six times. The numbers shown were 4, 5, 4, 3, 1, 4.

a) Determine the mean and the standard deviation of the six numbers shown.

A fair die is rolled four times and the four numbers shown are recorded one after the other to create a four-digit number.

b) What percentage of all such four-digit numbers contains at least two identical digits?

A fair die is rolled several times.

c) Determine the smallest integer value of n for which it is true that there will be at least 3 identical numbers shown when the die is rolled n times. You do not need to explain your answer.

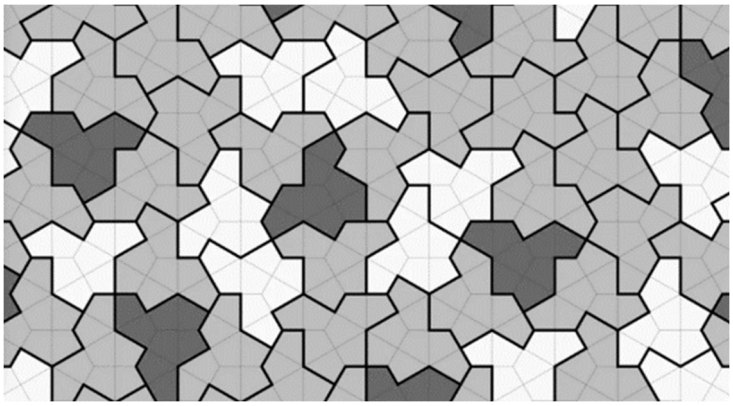
A fair die is rolled repeatedly, until there are 2 identical results among the numbers shown.

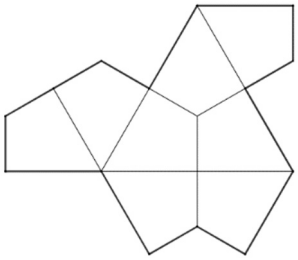
d) Determine the expected value of the number of dice rolls required to achieve this.

a)	3 points	
b)	5 points	
c)	2 points	
d)	6 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

6. In 2023, Englishman David Smith (a 64-year-old retired printing press mechanic) discovered the first single-unit, non-periodic tessellation that fully covers the plane, as shown in the diagram.[1] The 13-sided unit discovered by Smith can be assembled from 8 congruent kites and was given the name *'hat'*.





a) Prove that the angles of the kite are 60°, 90°, 120° and 90°.

b) Calculate the perimeter of one hat, given that its area is $\sqrt{1728}$ area units. Give an exact answer, without approximations.

A hat is coloured in three given colours, such that each kite is coloured in a single colour and adjacent kites may not be of the same colour. (Kites are adjacent if they share a common side.)

c) How many different ways are there to colour the eight kites forming one hat in the manner described above?

a)	3 points	
b)	7 points	
c)	6 points	
T.:	16 points	

[1] https://index.hu/techtud/2023/04/01/matematika-nemismetlodo-csempek-problema-penrose-einstein-nyugdijas-angol-smith-kalap-teknos/ (Last access: 09.09.2025.)
Tessellation is any particular way to cover a plane without gaps and overlaps. A tessellation is non-periodic if there does not exist a region of the plane that, by means of periodic repetition, would provide the pattern covering the infinity of the plane.

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

7. The operating costs of a shipping line consist of two components. Fuel costs are approximately proportional to the speed of the boat: cruising at a speed v km/h the cost is $1.2v$ ducats per kilometre. Other miscellaneous costs are proportional to operation time: 90 ducats for each hour of operation. The boat will travel a distance of 10 km.

a) Calculate the operating costs in ducats for this trip, given that the boat will cruise at a constant speed of 12 km/h.

b) Give the constant speed the boat should cruise at to minimise the operating costs for this 10 km trip. Give this minimal cost, too.

There were 50 passengers on a boat travelling from Révfülöp to Balatonboglár, each paying an average 1650 Ft for the journey. On the return trip, the same boat carried 70 passengers, paying an average fee of 1500 Ft each.

c) What was the average ticket price for the two trips combined?

a)	4 points	
b)	9 points	
c)	3 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

8. The quadratic function f is defined over the set of real numbers and has its zeros at –3 and 4. The graph of f is a parabola that passes through the y axis at the point (0; 6).

a) Determine the equation of this parabola.

The function $g(x) = 0.5x^2 - 2x - 6$ is defined over the set of real numbers.

b) A tangent is drawn to the graph of g at the point where $x = 4$. Determine the equation of the tangent.

c) Calculate the area of the region bounded by the graphs of the line $y = -2x + 2$ and the parabola $y = 0.5x^2 - 2x - 6$.

a)	5 points	
b)	4 points	
c)	7 points	
T.:	16 points	

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Write the number of the problem NOT selected in the blank square on page 2.

9. TAJ codes (social security numbers) are 9-digit identification codes with the first 8 characters being numerical digits, while the ninth one is a check digit for the first 8 digits. This check digit is created as follows: proceeding from left to right, digits at odd places are multiplied by 3 and those at even places are multiplied by 7. The last digit of the sum of these products will be the check digit.

a) Determine the check digit of a TAJ code the first 8 digits of which are 24165379.

b) The first digit of a TAJ code is covered, the rest of the code is _14564797. Determine the covered digit.

c) Determine all possible values of the check digit in a TAJ code that is in the form 02563*abba*. (Here *a* and *b* are not necessarily different digits.)

A school keeps records of TAJ codes. However (due to a machine error) 1.5% of the TAJ codes are recorded incorrectly. (This may be considered as a 0.015 probability for an arbitrary TAJ code to be recorded incorrectly.) 20 TAJ codes have been accessed in the registry.

d) Calculate the probability that there will be more than one among these codes that are recorded incorrectly.

a)	2 points	
b)	4 points	
c)	5 points	
d)	5 points	
T.:	16 points	

	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	13		**51**	
	2.	14			
	3.	12			
	4.	12			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date examiner

	pontszáma **egész számra** kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum dátum

javító tanár jegyző