ÉRETTSÉGI VIZSGA • 2025. október 14.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ
ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - conceptual error: *double underline*
   - calculation error or other, non-conceptual, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless otherwise stated in the answer key**. However, scores awarded must always be whole numbers.
3. In the event of a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a conceptual error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through a conceptual error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without the remark as well.

6. Deduct points for **missing units of measurement** only if the missing unit of measurement is part of the answer or unit conversion (without parentheses).
7. If there is more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the course of the elaboration of a particular solution **is acceptable without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, substitution of values provided in the tables of the *Négyjegyű függvénytáblázat* (Formula Booklet) (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, and finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1. a)

$78\cdot 3^{x-1}=26\cdot 3^{x}$, so the equation may be written as: $9\cdot 9^{x}+26\cdot 3^{x}-3=0$.	1 point	
Since $9^{x}=(3^{x})^{2}$, the equation is quadratic in 3^{x}.	1 point	*Award this point also if this is not explicitly stated but apparent from the solution.*
The solutions are –3 and $\frac{1}{9}$.	1 point	
The equation $3^{x}=-3$ does not have a solution.	1 point	
The solution of the equation $3^{x}=\frac{1}{9}$ is $x = -2$.	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**6 points**	

1. b)

Let q be the common ratio of the sequence. Then ($b_5=b_2\cdot q^3$, i.e.) $162 = 48\cdot q^3$.	1 point	
$q = 1.5$.	1 point	
$b_1=\frac{b_2}{q}=\frac{48}{1.5}=32$	1 point	$b_n=b_2\cdot q^{n-2}=48\cdot 1.5^{n-2}$ $48\cdot 1.5^{n-2}>10^7$
($b_n=32\cdot 1.5^{n-1}$, so) the inequality $32\cdot 1.5^{n-1}>10^7$ needs to be solved, which is equivalent to $1.5^{n-1}>312500$ ($n\in\mathbf{N}$).	1 point	
The base 1.5 logarithmic / exponential function is strictly monotone increasing,	1 point	
therefore $n-1>\log_{1.5}312\,500\approx 31.2$.	1 point	
$n > 32.2$ (i.e. all terms of the sequence with an index greater than 32 are greater than 10^7).	1 point	$n\geq 33$ ($n\in\mathbf{N}$)
Total:	**7 points**	

Notes:

1. Award a maximum of 6 marks if the candidate provides the correct answer by listing the terms of the sequence (with reasonable and correct rounding), but does not refer to strict monotonicity.

2. Award a maximum of 6 points if the candidate solves an equation instead of an inequality but does not cite strictly monotonic increase.

2. a) **Solution 1**		
Let x be the number of early bird tickets. In this case the number of tickets bought on-site will be $917 - x$.	1 point	
$2500x + 3000 \cdot (917 - x) = 2\,380\,000$	1 point	
$2500x + 2\,751\,000 - 3000x = 2\,380\,000$ $371\,000 = 500x$	1 point	
This gives $x = 742$ (early bird tickets were bought),	1 point	
and $(917 - 742 =)$ 175 tickets were bought on-site.	1 point	
Check: $2500 \cdot 742 + 3000 \cdot 175 = 2\,380\,000$	1 point	
Total:	**6 points**	

2. a) **Solution 2**		
Let x be the number of early bird tickets and let y be the number of tickets bought on-site. The following system must then be solved: $\begin{cases} x + y = 917 \\ 2500x + 3000y = 2\,380\,000. \end{cases}$	2 points	
Multiplying the first equation by 5 and dividing the second by 500 yields: $\begin{cases} 5x + 5y = 4585 \\ 5x + 6y = 4760. \end{cases}$	1 point	
Subtracting the first equation from the second yields: $y = 175$ (tickets were bought on-site).	1 point	
Therefore $x = (917 - 175 =)$ 742 (tickets were bought in advance).	1 point	
Check: $(742 + 175 = 917$ and$)$ $2500 \cdot 742 + 3000 \cdot 175 = 2\,380\,000$	1 point	
Total:	**6 points**	

2. b) **Solution 1**		
(Without considering the order of selection) there are $\binom{40}{2}$ (= 780) different ways to select 2 boats out of 40 (total number of cases).	1 point	*(Considering the order as well)* $40 \cdot 39$ (= 1560).
(There are 35 non-winners among the boats.) One winner and one non-winner boat may be selected in $\binom{5}{1}\binom{35}{1}$ (= 175) different ways (number of favourable cases).	1 point	$5 \cdot 35 + 35 \cdot 5$ (= 350)
The probability is $\frac{\binom{5}{1}\binom{35}{1}}{\binom{40}{2}} = \frac{175}{780} = \frac{35}{156} \approx 0.224$.	1 point	$\frac{350}{1560} \approx 0.224$
Total:	**3 points**	

2. b) **Solution 2**		
The probability of first pulling a winner boat and then a non-winner is $\frac{5}{40} \cdot \frac{35}{39}$ (≈ 0.112).	1 point	
The probability of first pulling a non-winner boat and then a winner is $\frac{35}{40} \cdot \frac{5}{39}$ (≈ 0.112).	1 point	
The probability is $\left(\frac{35}{40} \cdot \frac{5}{39} + \frac{35}{40} \cdot \frac{5}{39} =\right) \frac{35}{156} \approx 0.224$.	1 point	
Total:	**3 points**	

2. c)		
Without any ties there are 3! = 6 different orders.	1 point	
With a tie on places 1 and 2 there are three possible orders, based on who is third.	1 point	*In case of a two-way tie, there are 3 ways to select the two contestants, while the places for the tie (1-2 or 2-3) may be selected in 2 different ways.*
Likewise, with a two-way tie on places 2 and 3 there are three possible orders again.	1 point	
There is only 1 possible way for a three-way tie.	1 point	
There are a total of ($6 + 2 \cdot 3 + 1 =$) 13 different possible ways for the three contestants to finish.	1 point	
Total:	**5 points**	

Note: Award full score if the candidate provides the correct answer based on an organised list of possible orders.

3. a) Solution 1		
We provide a list all possible routes (e.g. based on the length of the routes). 3-long routes: *ABDF*, *ACDF*, *ACEF* (3 possibilities). 4-long routes: *ABCDF*, *ABCEF*, *ABDEF, ACBDF*, *ACDEF*, *ACEDF* (6 possibilities). 5-long routes: *ABCDEF*, *ABCEDF*, *ABDCEF*, *ACBDEF* (4 possibilities).	4 points	
There are a total (3 + 6 + 4 =) 13 possible routes.	1 point	
Total:	**5 points**	

Note: In case the candidate is listing possible routes, award points as follows: 1 point for 3–5 correct routes, 2 points for 6–8 correct routes, 3 points for 9–10 correct routes, 4 points for 11–12 correct routes.

3. a) Solution 2		
(Separation of cases, according as edge *CD* is part of the route or not.) Case I: *CD* is not part of the route. Starting with *AC* there are 4 possible routes (2 possibilities from *C* and 2 ways to finish routes *CE* and *CBD*).	1 point	
For reasons of symmetry, there are also 4 possible routes starting with *AB*.	1 point	
Case II: *CD* is part of the route. For *C*→*D* there are 2 ways to get to *C* (from *A* or *B*) and there are also 2 ways to finish from *D* (*DF* or *DEF*). This is $2 \cdot 2 = 4$ possible routes.	1 point	
For *D*→*C* there is only one way, *ABD*, to get to *D*, and also one way only to finish from *C* (*CEF*), so this only gives 1 possible route.	1 point	
The total number of possible routes is then $2 \cdot 4 + 4 + 1 = 13$.	1 point	
Total:	**5 points**	

3. a) Solution 3		
(Separation of cases based on whether *A* will be followed by *B* or *C*.) Case I: Starting with *ABC* there will be 4 possible routes (2 ways to finish from both *E* and *D*).	1 point	
Case II: Starting with *ABD* there will be 3 possible routes (we may continue towards *C*, *E* or *F*).	1 point	
Case III: Starting with *ACB* we must continue to *D* and finish in 2 possible ways (to *E* or *F*).	1 point	
Case IV: Starting with *ACD* or *ACE* there will be 2 possible ways to finish (*E* or *F* in one case, *D* or *F* in the other).	1 point	
The total number of possible routes is then $4 + 3 + 2 + 2 \cdot 2 = 13$.	1 point	
Total:	**5 points**	

3. b) **Solution 1**		
$a^2+2ab+b^2-c^2=(a+b)^2-c^2=$	1 point	
$=(a+b+c)(a+b-c)$	1 point	
The triangle-inequality will guarantee that $a+b>c$ is true,	1 point	
therefore both factors of the product are positive, which makes the statement true.	1 point	
Total:	**4 points**	

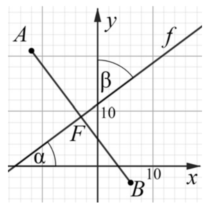
3. b) **Solution 2**		
From the Cosine Rule: $a^2+b^2-c^2=2ab\cos\gamma$,	1 point	
$a^2+2ab+b^2-c^2=2ab+2ab\cos\gamma=2ab(1+\cos\gamma)$.	1 point	
In any triangle $-1<\cos\gamma<1$ is true,	1 point	
all factors of the product are positive, which makes the statement true.	1 point	
Total:	**4 points**	

3. b) **Solution 3**		
Rearrangement of the inequality to be proven yields: $a^2+2ab+b^2>c^2$, that is $(a+b)^2>c^2$.	1 point	
As both $a+b$ and c are positive, $a+b>c$,	1 point	
which is guaranteed by the triangle-inequality.	1 point	
Only equivalent operations were performed, the original statement is therefore true.	1 point	
Total:	**4 points**	

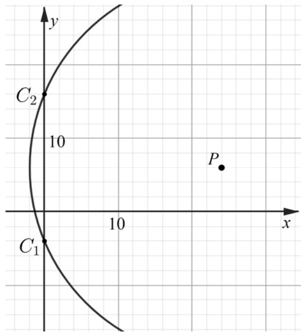
3. c)		
The converse of the statement: "If (for the positive real numbers a, b and c) it is true that $a^2+b^2+2ab-c^2>0$, then there exists a triangle whose sides (in cm) are a, b and c long."	1 point	
The converse is false.	1 point	
Any counterexample (e.g. $a=2$, $b=1$, $c=1$).	1 point	
Total:	**3 points**	

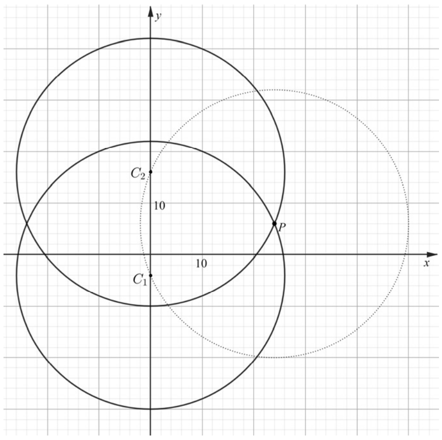
4. a)		
The coordinates of the midpoint F of segment AB are (–3; 9).	1 point	
$\overrightarrow{AB} = (18; -24)$, so one possible normal vector of f is (3; –4).	1 point	*the slope of f is* $\frac{3}{4}$.
f: $3x - 4y = (3 \cdot (-3) - 4 \cdot 9 =) -45$.	1 point	$f\colon\ y - 9 = \frac{3}{4}(x+3)$ $\left(y = \frac{3}{4}x + \frac{45}{4}\right)$
Let α be the angle between the line f and the x-axis. Then $\tan\alpha = \frac{3}{4}$,	1 point*	
and α ≈ 36.9°.	1 point*	
The angle between the line f and the y-axis is the complementary angle of α, which is about 53.1°.	1 point*	
Total:	**6 points**	

*Note: The 3 points marked * may also be given for the following reasoning:*

The intercepts of f on the axes are (–15; 0) and (0; 11.25).	1 point	
Let β be the angle between the line f and the y-axis, then $\tan\beta = \frac{15}{11.25}$,	1 point	
and β ≈ 53.1°.	1 point	

4. b) Solution 1		
Let $C(0; v)$ be the centre of the circle. Then the equation of the circle is $x^2 + (y - v)^2 = 26^2$.	1 point	*CP* = 26, *so* $\sqrt{24^2 + (6-v)^2} = 26$.
(As the circle passes through P) $24^2 + (6 - v)^2 = 26^2$.	1 point	
This means $(6 - v)^2 = 100$, therefore $6 - v = 10$ or $6 - v = -10$.	1 point	$v^2 - 12v - 64 = 0$
$v = -4$ or $v = 16$ (i.e. $C_1(0; -4)$ or $C_2(0; 16)$).	1 point	
The equation of the circle is $x^2 + (y + 4)^2 = 26^2$ or $x^2 + (y - 16)^2 = 26^2$.	2 points	
Total:	**6 points**	

4. b) **Solution 2**		
The centre C of the circle is a point of intersection of the y-axis and the circle centred at P with a radius of 26 units. The equation of such circle is $(x-24)^2+(y-6)^2=26^2$.	1 point	
(Computing the points of intersection of the y-axis and this circle:) If $x = 0$ then $24^2+(y-6)^2=26^2$.	1 point	
Then $(y-6)^2=100$, therefore $y - 6 = -10$ or $y - 6 = 10$.	1 point	
$y = -4$ or $y = 16$ (i.e. $C_1(0; -4)$ or $C_2(0; 16)$).	1 point	
The equation of the circle is $x^2+(y+4)^2=26^2$ or $x^2+(y-16)^2=26^2$.	2 points	
Total:	**6 points**	



II.

5. a)		
The mean of the 6 rolls is $\left(\frac{4+5+4+3+1+4}{6}=\right)3.5$.	1 point	*Award these points if the candidate correctly determines the mean and the standard deviation with a calculator.*
The standard deviation is $\sqrt{\frac{1.5^2+4\cdot0.5^2+2.5^2}{6}}=\sqrt{\frac{19}{12}}\approx1.26$.	2 points	
Total:	**3 points**	

5. b)		
There are a total of $6^4 = 1296$ different four-digit numbers.	1 point	
Use complementary counting.	1 point*	*Award this point also if this is not explicitly stated but apparent from the solution.*
Numbers not meeting the condition have four different digits. There are $6\cdot5\cdot4\cdot3 = 360$ such numbers.	1 point*	

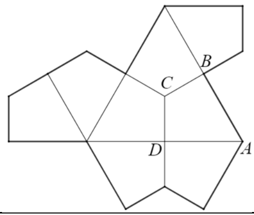
Therefore, the number of cases satisfying the condition is (1296 – 360 =) 936.	1 point*	
(As $\frac{936}{1296} \approx 0.722$) this is 72.2 percent of all possible such numbers.	1 point	
Total:	**5 points**	

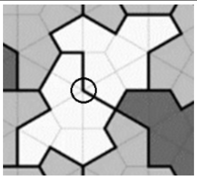
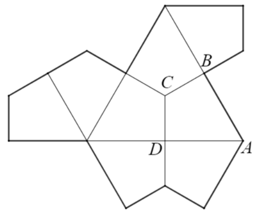
*Note: The 3 points marked * may also be given for the following reasoning:*

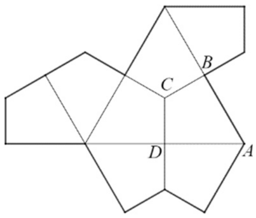
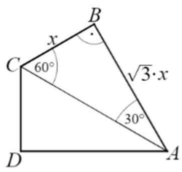
There are 6 numbers with four identical digits. There are $\binom{4}{3}\cdot 6\cdot 5 = 120$ numbers with exactly three identical digits. There are $\binom{4}{2}\cdot 6\cdot 5\cdot 4 = 720$ numbers with exactly two identical digits. There are $\binom{4}{2}\cdot\binom{6}{2} = 90$ numbers with two pairs of identical digits.	2 points	
There are (6 + 120 + 720 + 90 =) 936 appropriate numbers in total.	1 point	

5. c)		
$n = 13$ (There are 6 possible outcomes on a die. The pigeonhole principle states that there will certainly be at least 3 identical outcomes among 6·2 + 1 = 13 trials.)	2 points	
Total:	**2 points**	

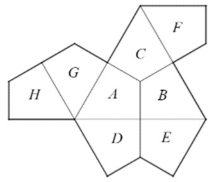
5. d)		
We'll need a minimum of 2 and a maximum of 7 rolls to certainly obtain at least two identical results.	1 point	*Award this point also if this is not explicitly stated but apparent from the solution.*
Let $P(n)$ be the probability that n rolls are needed. (The first number shown may be anything.) $P(2) = \frac{1}{6}$, $P(3) = \frac{5}{6}\cdot\frac{2}{6} = \frac{5}{18}$, $P(4) = \frac{5}{6}\cdot\frac{4}{6}\cdot\frac{3}{6} = \frac{5}{18}$, $P(5) = \frac{5}{6}\cdot\frac{4}{6}\cdot\frac{3}{6}\cdot\frac{4}{6} = \frac{5}{27}$, $P(6) = \frac{5}{6}\cdot\frac{4}{6}\cdot\frac{3}{6}\cdot\frac{2}{6}\cdot\frac{5}{6} = \frac{25}{324}$, $P(7) = \frac{5}{6}\cdot\frac{4}{6}\cdot\frac{3}{6}\cdot\frac{2}{6}\cdot\frac{1}{6}\cdot 1 \left(= 1 - \sum_{i=2}^{6} P(i)\right) = \frac{5}{324}$.	3 points	
The expected value of the number of times the die is rolled is $\sum_{i=2}^{7} P(i)\cdot i = \frac{1223}{324} \approx 3.775$.	2 points	
Total:	**6 points**	

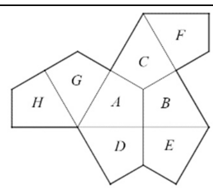
6. a) **Solution 1**		
The angle at vertex C of the kite $ABCD$ is $360° : 3 = 120°$.	1 point	
The angles at vertices B and D are $360° : 4 = 90°$.	1 point	
The angle at vertex A is $(360° – 120° – 2·90° =)\ 60°$.	1 point	
Total:	**3 points**	

6. a) **Solution 2**		
Six instances of the smallest angle combined will make 360°, so each must be 60°. 	1 point	
The angles at vertices B and D are $360° : 4 = 90°$.	1 point	
The angle at vertex C is $(360° – 60° – 2·90° =)\ 120°$.	1 point	
Total:	**3 points**	

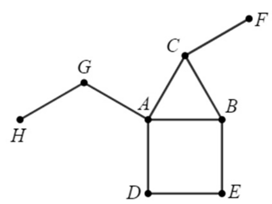
6. b)		
The perimeter of the hat consists of six segments with the length of AB and eight segments with the length of BC ($k = 6·AB + 8·BC$). 	1 point	*Award this point also if this is not explicitly stated but apparent from the solution.*
The area of one kite is $\frac{\sqrt{1728}}{8}(=3\sqrt{3})$.	1 point	
Diagonal AC divides the kite $ABCD$ into two congruent triangles, each of them half of a regular triangle. If the length of side BC is x, then the length of side AB is $\sqrt{3}\cdot x$. 	1 point	$\frac{AB}{BC}=\tan 60°=\sqrt{3}$, *if* $BC = x$, *then* $AB = \sqrt{3}\cdot x$.
The area of the kite is twice the area of a half regular triangle, therefore $3\sqrt{3}=2\cdot\frac{x\cdot\sqrt{3}\cdot x}{2}$,	2 points	
whence $x^2 = 3$ and $x = \sqrt{3}$.	1 point	
The perimeter of the hat is therefore $6\cdot\sqrt{3}\cdot x+8\cdot x=18+8\sqrt{3}$.	1 point	
Total:	**7 points**	

Note: Award a maximum of 6 points if the candidate uses any approximate values.

6. c) Solution 1		
The kites marked *A*, *B* and *C* in the diagram must be coloured in three different colours.	1 point	
This may be done in 3! = 6 different ways.	1 point	
There are 2 colour options for each of *F*, *G* and *H* (as *F* is different from *C*, *G* is different from *A* and *H* is different from *G*).	1 point	
Depending on the colour of *D* there are two possible cases. Case I: if *D* and *B* are of the same colour then *E* could be one of 2 colours (which is different from the shared colours of *B* and *D*). This gives 2 possibilities.	1 point	
Case II: If *D* and *B* are of different colours then there is only 1 possible colour for *E* (same as that of *A*). Only 1 possibility in this case.	1 point	
Altogether, there are $6\cdot 2^3\cdot (2+1) = 144$ different ways to colour the hat.	1 point	
Total:	**6 points**	

6. c) Solution 2		
Kite *F* can be one of three different colours, then kite *C* may be one of 2 different colours.	1 point	
Kites *A*, *B* and *C* must have different colours, so there are 2 options for the colour of *A* (different from *C*), which also determines the one colour *B* may have.	1 point	
Depending on the colour of *D* there are two possible cases. Case I: if *D* and *B* are of the same colour then *E* could be one of 2 colours (which is different from the shared colours of *B* and *D*). This gives 2 possibilities.	1 point	
Case II: If *D* and *B* are of different colours then there is only 1 possible colour for *E* (same as that of *A*). Only 1 possibility in this case.	1 point	
G and *H* may have 2 different colours each (*G* is different from *A*, *H* is different from *G*).	1 point	
Altogether, there are $3\cdot 2\cdot 2\cdot (2+1)\cdot 2^2 = 144$ different ways to colour the hat.	1 point	
Total:	**6 points**	

Note: The kites making up the hat may be represented by the 8 vertices of a graph. The edges of the graph represent adjacency. The vertices of the graph must then be coloured such that the two endpoints of each edge should be of different colours.



7. a)		
The cost of fuel on a 10 km trip, cruising at 12 km/h is $10 \cdot 1.2 \cdot 12 = 144$ ducats.	1 point	
Cruising at 12 km/h, the boat completes the 10 km trip in $\frac{10}{12}$ hours.	1 point	
Other miscellaneous costs will amount to $\frac{10}{12} \cdot 90 = 75$ ducats.	1 point	
Cruising at 12 km/h, the full operating cost for the 10 km trip will be (144 + 75 =) 219 ducats.	1 point	
Total:	**4 points**	

7. b)		
The cost of fuel on a 10 km trip, cruising at v km/h is $10 \cdot 1.2 \cdot v$ ducats.	1 point	
Cruising at v km/h, the boat completes the 10 km trip in $\frac{10}{v}$ hours.	1 point	
Other miscellaneous costs will amount to $\frac{10}{v} \cdot 90 = \frac{900}{v}$ ducats.	1 point	
Cruising at v km/h, the full operating cost for the 10 km trip will be $12v + \frac{900}{v}$ ducats.	1 point	
The function f: $\mathbf{R}^+ \to \mathbf{R}$, $f(v) = 12v + \frac{900}{v}$ may have a minimum wherever the first derivative is zero.	1 point*	*Award this point also if this is not explicitly stated but apparent from the solution.*
$f'(v) = 12 - \frac{900}{v^2} = 0$	1 point*	
Then $v = \sqrt{\frac{900}{12}} = \sqrt{75} \approx 8.66$ km/h (as $v > 0$).	1 point*	
The value of the function f' is negative where $v < \sqrt{75}$, and positive where $v > \sqrt{75}$. Hence, $\sqrt{75}$ is an (absolute) minimum of f, indeed.	1 point*	$f''(v) = \frac{1800}{v^3} > 0$
The full operating cost on the 10 km trip will be minimal when the boat is cruising at 8.66 km/h, it will be approximately 208 ducats.	1 point	
Total:	**9 points**	

*The points marked * may also be given for the following reasoning:*

By the inequality between arithmetic and geometric means (of positive numbers) $12v+\frac{900}{v}\geq 2\cdot\sqrt{12v\cdot\frac{900}{v}}=120\sqrt{3}\approx 207.8$.	2 points	
Equality (representing minimal operating costs) will occur for $12v=\frac{900}{v}$.	1 point	
Hence $v^2 = 75$, $v = 5\sqrt{3}$ (as $v > 0$).	1 point	

7. c)		
Passengers paid a total of $50\cdot 1650$ (= 82 500) Ft on the first trip,	1 point	
and a total of $70\cdot 1500$ (=105 000) Ft on the second.	1 point	
The average ticket cost for the two trips combined was $\frac{82\,500+105\,000}{50+70}=1562.5$ Ft.	1 point	
Total:	**3 points**	

8. a) Solution 1		
The equation of the parabola can be stated in the (general) form of $y = ax^2 + bx + c$. $f(0) = 6$, therefore $c = 6$. $f(-3) = 0$ and $f(4) = 0$, therefore $9a - 3b + 6 = 0$ and $16a + 4b + 6 = 0$.	2 points	
From the first equation of the system $\begin{cases} 9a-3b+6=0 \\ 16a+4b+6=0 \end{cases}$ $b = 3a + 2$.	1 point	
Substituting into the second equation: $28a + 14 = 0$, then $a = -0.5$ and $b = 3\cdot(-0.5) + 2 = 0.5$.	1 point	
The equation of the parabola is $y = -0.5x^2 + 0.5x + 6$.	1 point	
Total:	**5 points**	

8. a) Solution 2		
Using the stated zeros of f the equation of the parabola may be written in the form of $y = a(x + 3)(x - 4)$.	2 points	
$f(0) = 6$, therefore $6 = -12a$, so $a = -0.5$.	2 points	
The equation of the parabola is $y = -0.5(x + 3)(x - 4)$.	1 point	
Total:	**5 points**	

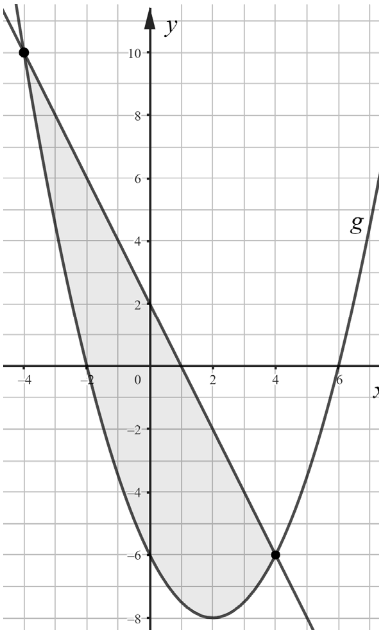
8. a) **Solution 3**		
The equation of the parabola can be stated in the (general) form of $y = ax^2 + bx + c$. (As the real roots of the quadratic equation $ax^2 + bx + c = 0$ are given as –3 and 4) Viéte's Formulae will apply as follows: the sum of the two roots is $-\frac{b}{a} = 1$, the product of the two roots is $\frac{c}{a} = -12$.	2 points	
$f(0) = 6$, so $c = 6$.	1 point	
Substitution into the second equation yields $a = -0.5$, and substituting that into the first equation gives $b = 0.5$.	1 point	
The equation of the parabola is $y = -0.5x^2 + 0.5x + 6$.	1 point	
Total:	**5 points**	

8. a) **Solution 4**		
State the equation of the parabola in the (vertex) form $y = a(x - u)^2 + v$, where the value of u is the arithmetic mean of the zeros: $u = \frac{-3+4}{2} = 0.5$.	2 points	
$f(0) = 6$, so $0.25a + v = 6$, and $f(4) = 0$, so $12.25a + v = 0$.	1 point	
Subtracting the equations will give $12a = -6$, so $a = -0.5$, and so $v = 6 - 0.25\cdot(-0.5) = 6.125$.	1 point	
The equation of the parabola is $y = -0.5(x - 0.5)^2 + 6.125$.	1 point	
Total:	**5 points**	

8. b)		
$g(4) = -6$, therefore the point of tangency is $E(4; -6)$.	1 point	
$g'(x) = x - 2$,	1 point	
therefore the gradient of the tangent line is $g'(4) = 2$.	1 point	
The equation of the tangent line is $y + 6 = 2(x - 4)$.	1 point	$y = 2x - 14$
Total:	**4 points**	

8. c)		
The first coordinates of the points of intersections of the two graphs will be given by the solutions of the equation $-2x + 2 = 0.5x^2 - 2x - 6$.	1 point	
Rearranged: $x^2 = 16$,	1 point	
the solutions of which are –4 and 4.	1 point	

(Over the interval [–4; 4] the line is 'above' the parabola) $A=\int_{-4}^{4}(-2x+2-0.5x^2+2x+6)\mathrm{d}x=\int_{-4}^{4}(-0.5x^2+8)\mathrm{d}x=$	1 point	
$=\left[\frac{-0.5x^3}{3}+8x\right]_{-4}^{4}=$	1 point	
$=\left(\frac{-32}{3}+32\right)-\left(\frac{32}{3}-32\right)=$	1 point	
$=\frac{128}{3}$. (The area of the region bounded by the graphs is $\frac{128}{3}\approx 42.67$ area units.)	1 point	
Total:	**7 points**	



9. a)		
$3\cdot(2+1+5+7)+7\cdot(4+6+3+9)=199$	1 point	
The check digit is 9.	1 point	
Total:	**2 points**	

9. b)		
Let x be the first digit. Then $3\cdot(x+4+6+7)+7\cdot(1+5+4+9)=$	1 point	
$=3x+184$.	1 point	
Based on the check digit, this number must end in 7, so $3x$ must end in 3.	1 point	
This is only possible if $x=1$(as x is a digit).	1 point	
Total:	**4 points**	

9. c)		
$3\cdot(0+5+3+b)+7\cdot(2+6+a+b)=$	1 point	
$=80+7a+10b$	1 point	
(As $10b+80$ ends in 0 and the value of the check digit is a) the last digit of $7a$ is a.	1 point	
This is possible, for $a=0$,	1 point	
or $a=5$.	1 point	
Total:	**5 points**	

Note: In both cases, the value of b is arbitrary.

9. d) Solution 1		
The probability that a TAJ code is correct is 0.985.	1 point	*Award this point also if this is not explicitly stated but apparent from the solution.*
The probability that all of 20 TAJ codes are correct is $0.985^{20}\approx 0.739$.	1 point	
The probability that exactly one of the 20 TAJ codes is incorrect is $20\cdot 0.985^{19}\cdot 0.015\approx 0.225$.	2 points	
The probability in question (using complements) is $1-0.739-0.225\approx 0.036$.	1 point	
Total:	**5 points**	

9. d) Solution 2		
The probability that a TAJ code is correct is 0.985.	1 point	*Award this point also if this is not explicitly stated but apparent from the solution.*
Let $P(i)$ be the probability that exactly i of the 20 TAJ codes are incorrect. $P(2)=\binom{20}{2}\cdot 0.985^{18}\cdot 0.015^{2}\approx 0.0326$ $P(3)=\binom{20}{3}\cdot 0.985^{17}\cdot 0.015^{3}\approx 0.0030$ $P(4)=\binom{20}{4}\cdot 0.985^{16}\cdot 0.015^{4}\approx 0.0002$	2 points	
(As $P(5)=\binom{20}{5}\cdot 0.985^{15}\cdot 0.015^{5}\approx 0{,}00001$,) the probability that more than 4 of the 20 TAJ codes are incorrect is negligible.	1 point	
The probability in question is $(0.0326+0.0030+0.0002\approx)\ 0.036$.	1 point	
Total:	**5 points**	