ÉRETTSÉGI VIZSGA • 2020. október 20.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$A \cap B = \{1; 10\}$	1 point	
$A \setminus B = \{3; 6; 15\}$	1 point	
Total:	**2 points**	

2.		
She is running a total 7000 m (35 laps) during these five days.	2 points	
Total:	**2 points**	

3.		
$x = 8$	2 points	
Total:	**2 points**	

4.		
2^{101}	2 points	*Not to be divided.*
Total:	**2 points**	

5.		
A: false B: true C: true	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

6.		
a) $f(12) = 1000$	1 point	
b) $\left(\frac{x}{4} = 2, \text{ i.e.}\right)$ $x = 8$	2 points	
Total:	**3 points**	

7.		
At the end of October the price was raised to $15\,000 \cdot 1.25 = 18\,750$ forints.	1 point	*1.25x, where x is the original price.*
$\frac{15000}{18750} \cdot 100 = 80\%$	1 point	$\frac{x}{1.25x} = 0.8$
At the end of November the discount was 20%.	1 point	
Total:	**3 points**	

8. Solution 1		
The surface area of a cube with an edge of length b is $A = 6b^2$, so $b = 1.5$ (cm).	1 point	
The edge of a cube that is twice that size is 3 (cm),	1 point	
and so its surface area is $(6 \cdot 3^2 =)$ 54 cm².	1 point	
Total:	**3 points**	

8. Solution 2		
The two dice are similar, the ratio of similarity is 2 : 1.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The area of similar solids varies with the square of the ratio of similarity, so the surface area of the larger cube is four times that of the smaller one,	1 point	
that is 54 cm².	1 point	
Total:	**3 points**	

9.		
$\left(\frac{6!}{2!\cdot 4!}=\right)15$	2 points	
Total:	**2 points**	

10.		
a) [–1; 3]	2 points	
b) –1 and 1	2 points	
Total:	**4 points**	

11.		
The mean is 30 (minutes).	1 point	
The standard deviation is $\sqrt{\frac{(38-30)^2+(30-30)^2+2\cdot(26-30)^2}{4}} =$	1 point	*Award this point if the candidate calculates the correct standard deviation with a calculator.*
$= \sqrt{24} \approx 4.9$ (minutes).	1 point	
Total:	**3 points**	

12.		
$\frac{30}{36}=\frac{5}{6}$	2 points	
Total:	**2 points**	

II. A

13. a)		
Let x be the number I thought of. In this case $\left(\frac{x}{2}-5\right)\cdot 4+8=x$.	2 points	
Removing the brackets: $2x - 20 + 8 = x$.	1 point	
Rearranged: $x = 12$, this is the number I thought of.	1 point	
Check against the original text: half of 12 is 6, take away 5 gives 1. Multiply this by 4 and add 8, the result is 12.	1 point	
Total:	**5 points**	

Note: Award a total of 2 points if the candidate gives the correct answer and checks it without any reasoning.

13. b) Solution 1		
Let a be the first term of the sequence, a d be the common difference. The following system is to be solved: $\left.\begin{array}{r} a+9d=18 \\ a+29d=48 \end{array}\right\}$.	2 points	
Express a from the first equation and substitute it into the second: $18 - 9d + 29d = 48$.	1 point	*Subtract the first equation from the second:* $20d = 30$.
$d = 1.5$,	1 point	
$a = 4.5$.	1 point	
Total:	**5 points**	

13. b) Solution 2		
As per the general properties of an arithmetic sequence, the 30th term of the sequence is 20 times the common difference greater than the 10th term: $a_{30} = a_{10} + 20d$.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The common difference is therefore $d = (48 - 18):20 = 1.5$.	2 points	
The first term: $a_1 = 18 - 9\cdot 1.5 = 4.5$.	2 points	
Total:	**5 points**	

14. a)		
Use the Pythagorean theorem: $AC^2 + 40^2 = 41^2$,	1 point	
here $AC = 9$ (cm).	1 point	
The area of the triangle is $A = \frac{9 \cdot 40}{2} =$	1 point	
$= 180\ \text{cm}^2$,	1 point	
that is $1.8\ \text{dm}^2$.	1 point	
Total:	**5 points**	

14. b)		
Let α be the angle at vertex *A* of the triangle. $\sin\alpha = \frac{40}{41}$,	1 point	$\cos\alpha = \frac{9}{41}$
$\alpha \approx 77.32°$.	1 point	
The angle at vertex *B* is $\beta = 90° - \alpha = 12.68°$.	1 point	
Total:	**3 points**	

14. c)		
(Use the converse of Thales' Theorem) the centre of the circumcircle of a right triangle is the midpoint of the hypotenuse.	1 point*	*This point is also due if the correct reasoning is reflected only by the solution.*
The diameter of the circumcircle of the triangle is therefore $d = 41$ (cm).	1 point*	*The radius is* $r = 20.5$ (cm).
The perimeter of the circle is $P = d \cdot \pi = 41\pi \approx$	1 point	$K = 2r \cdot \pi$
≈ 129 m rounded to the nearest cm.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**4 points**	

*Note: The points marked * are due if the candidate calculates the radius of the circumcircle using other formulae, such as* $A = \frac{abc}{4r}$ *or* $a = 2r \cdot \sin\alpha$.

15. a)		
For the 1998 data $x = 98$, for 2018: $x = 118$	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$f(98) = 0.0001 \cdot 98^2 - 0.0063 \cdot 98 + 15.2 = 15.543$	1 point	
$f(118) = 0.0001 \cdot 118^2 - 0.0063 \cdot 118 + 15.2 = 15.849$	1 point	
The 2018 annual mean temperature was $(15.849 - 15.543 \approx)$ 0.3 °C greater than that in 1998.	1 point	
Total:	**4 points**	

15. b)		
The equation $0.0001x^2 - 0.0063x + 15.2 = 15.42$ is to be solved $(0 \le x \le 119)$.	1 point	
Rearranged: $0.0001x^2 - 0.0063x - 0.22 = 0$.	1 point	$x^2 - 63x - 2200 = 0$
The solutions are: $x_1 = -25$, $x_2 = 88$.	1 point	
–25 is not a proper solution (while 88 is, as $0 \le 88 \le 119$).	1 point	
The annual mean temperature was 15.42 °C in the year (1900 + 88 =) 1988.	1 point	
Total:	**5 points**	

15. c)		
The equation $15.92 \cdot 1.002^t = 16.7$ is to be solved $(0 \le t)$.	1 point	
Rearranged: $1.002^t \approx 1.049$.	1 point	
$t = \log_{1.002} 1.049 \left(= \frac{\lg 1.049}{\lg 1.002} \right) \approx 23.94$	2 points	
According to this model, the annual mean temperature will be 16.7 °C in the year (2018 + 24 =) 2042.	1 point	
Total:	**5 points**	

Note: Award full credit if the candidate gives the correct answer by calculating the annual mean temperatures year by year, rounding it reasonably (to at least two decimal places).

II. B

16. a)		
365.25 days = 24 · 365.25 = 8766 hours	1 point	
The average speed of Earth for one full orbit is $v = \frac{s}{t} = \frac{939\,000\,000}{8766} \approx$	1 point	
$\approx$ 107 118 km/h.	1 point	
Total:	**3 points**	

16. b)		
4.2 hours = (4.2 · 60 · 60 =) 15 120 seconds	1 point	
During this much time light travels 15 120·300 000 =	1 point	
$(= 1.512\cdot 10^4\cdot 3\cdot 10^5 \approx)$ $4.5\cdot 10^9$ km, that is the approximate distance between Neptune and the Sun.	1 point	
Total:	**3 points**	

16. c)		
There is only 1 correct order for the planets.	1 point	
Mercury and Venus may be placed in two different orders, while the other for planets may be placed in 4! = 24 different orders.	1 point	
This gives a total 2 · 24 = 48 possibilities (the total number of cases).	1 point	
The probability is $\frac{1}{48}\approx 0.021$.	1 point	
Total:	**4 points**	

16. d) Solution 1		
In case of selection without replacement (disregarding the order of selection) the number of favourable cases is 7, as there are seven other planets besides Earth.	1 point	*(Taking the order of selection into account) there are* 2 · 7 = 14 *favourable outcomes.*
The total number of cases is $\binom{8}{2}=28$.	1 point	*total number of cases:* $8\cdot 7=56$.
The probability of the event in this case is $\frac{7}{28}=\frac{1}{4}=0.25$.	1 point	
In case of selection with replacement the total number of cases is $8^2 = 64$.	1 point	
Favourable cases: both cards read "Earth": 1 case or one card reads "Earth", the other does not: 2·7 = 14 cases, 15 cases altogether.	1 point	*The number of unfavourable cases is* 7·7 = 49.
The probability in this case is $\frac{15}{64}(\approx 0.234)$.	1 point	$1-\frac{49}{64}=\frac{15}{64}$
The probability that the name of planet Earth will appear at least once is therefore greater in case of selection without replacement.	1 point	
Total:	**7 points**	

16. d) **Solution 2**		
In case of selection without replacement the probability that the first card reads “Earth” is $\frac{1}{8}$.	1 point	
The probability that the first card does not read “Earth” but the second card does is: $\frac{7}{8}\cdot\frac{1}{7}=\frac{1}{8}=0.125$.	1 point	
(As these are mutually exclusive events) the total probability is the sum of the above: $\frac{2}{8}$.	1 point	
In case of selection with replacement the probability that the first card reads “Earth” is $\frac{1}{8}$.	1 point*	*The probability that neither card reads “Earth” is* $\frac{7}{8}\cdot\frac{7}{8}=\frac{49}{64}$.
The probability that the first card does not read “Earth” but the second card does is $\frac{7}{8}\cdot\frac{1}{8}=\frac{7}{64}$.	1 point*	
The total probability in case of selection with replacement is $\frac{1}{8}+\frac{7}{64}\approx 0.234$.	1 point*	*The probability that at least one card reads “Earth” is* $1-\frac{49}{64}=\frac{15}{64}$.
The probability that the name of planet Earth will appear at least once is therefore greater in case of selection without replacement.	1 point	
Total:	**7 points**	

Notes:

*1. The points marked * may be awarded for the following reasoning:*

In case of selection with replacement the probability that the first or the second card reads “Earth” is $\frac{1}{8}+\frac{1}{8}$	1 point	
although the case when both cards read “Earth” was counted twice here. The probability of this is $\frac{1}{8}\cdot\frac{1}{8}=\frac{1}{64}$	1 point	
The total probability in case of selection with replacement is $\frac{1}{8}+\frac{1}{8}-\frac{1}{64}\approx 0.234$.	1 point	

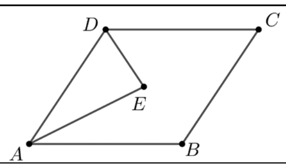
2. Award full credit for the following reasoning:

Both cases may be considered as drawing twice, one after the other, from among the cards. If the first card reads “Earth”, than it does not matter any more whether the card has been replaced or not.

If the first card does not read “Earth” than, obviously, we will have a better chance of pulling Earth for the second time when the first card is not replaced, as there will be fewer cards to select from that we do not want to see.

The probability that the name of planet Earth will appear at least once is therefore greater in case of selection without replacement.

17. a)

A proper graph, for example: 	2 points	*Accept graphs that are not simple.*
Total:	**2 points**	

17. b)

If such a graph existed, the sum of all degrees would be (3·5 =) 15.	1 point	
As the sum of all degrees of a graph must be even	1 point	
this graph does not exist.	1 point	
Total:	**3 points**	

17. c)

$\overrightarrow{AD} = -\overrightarrow{DA}$	1 point*	$\overrightarrow{AB} = \overrightarrow{DB} - \overrightarrow{DA}$
$\overrightarrow{DB} = 2 \cdot \overrightarrow{DE}$	1 point*	
$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB} = -\overrightarrow{DA} + 2 \cdot \overrightarrow{DE}$	1 point	$\overrightarrow{AB} = 2 \cdot \overrightarrow{DE} - \overrightarrow{DA}$
Total:	**3 points**	

*Note: The 2 points marked * are also due if the correct reasoning is reflected only by the solution.*

17. d)

Substitute the first coordinate of point *B* into the equation of line *AB*: 2·3 – 5*y* = –4,	1 point	
that gives *y* = 2, so *B*(3; 2) is one of the vertices.	1 point	
(Point *A* is the point of intersection of lines *AB* and *AD*, its coordinates are given as the solution of the system formed by the equations of the above lines.) Express *x* from the first equation and substitute it into the second: $3 \cdot \frac{5y-4}{2} - 2y = -6$.	2 points	*Multiply the first equation by 3 and the second by 2 and subtract:* –11y = 0.
Rearranged: *y* = 0.	1 point	
From this *x* = –2, so *A*(–2, 0) is the other vertex.	1 point	
(Segments *AC* and *BD* have a common midpoint *E*.) The midpoint of segment *AC* is *E*(1.5; 2.5).	1 point	$\overrightarrow{BC} = \overrightarrow{AD} = (2;3)$
As $1.5 = \frac{d_1+3}{2}$ and $2.5 = \frac{d_2+2}{2}$, the coordinates of point $D(d_1; d_2)$ are (0; 3).	2 points	*The position vector of point D is* (–2; 0) + (2; 3) = (0; 3), *these are also the coordinates of point D.*
Total:	**9 points**	

18. a)

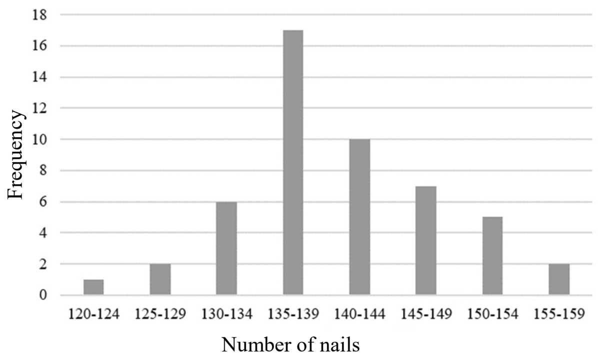
The full length of a nail is 1 + 25 + 2.5 = 28.5 mm.	2 points	
Total:	**2 points**	

18. b)

(Calculate the volume of a single nail.) The volume of the head of the nail is: $V_1 = \frac{1 \cdot \pi}{3} \cdot (2.5^2 + 2.5 \cdot 1 + 1^2) = 3.25\pi \approx 10.21$ (mm^3).	1 point	
The volume of the cylindrical body: $V_2 = 1^2 \cdot \pi \cdot 25 = 25\pi \approx 78.54$ (mm^3).	1 point	
The volume of the tip: $V_3 = \frac{1^2 \cdot \pi \cdot 2.5}{3} = \frac{5}{6}\pi \approx 2.62$ (mm^3).	1 point	
The total volume of a single nail is the sum of the above, $V \approx 91.37$ mm^3.	1 point	
7,8 g/cm^3 = 0.0078 g/mm^3	1 point	91.37 mm^3 =0.09137 cm^3
The mass of a single nail is $91.37 \cdot 0.0078 = 0.713$ g.	1 point	$0.09137 \cdot 7.8$
10 dkg = 100 g	1 point	
100 grams of nails is $\frac{100}{0.713} \approx 140$ nails.	1 point	
Total:	**8 points**	

Note: Deduce a total 1 point if the candidate consequently works with the given diameter as radius, but otherwise calculates correctly.

18. c)

	3 points	*Award 1 point for the correct notation,* *2 points for the proper heights of the bars.*
Total:	**3 points**	

18. d)

The table showing the class averages:

Number of nails	Frequency	Number of nails	Frequency
122	1	142	10
127	2	147	7
132	6	152	5
137	17	157	2

(table above)	1 point	*Award this point if the candidate correctly cal-culates without the table shown.*
The median (the mean of the 25th and 26th data) is 137 nails,	1 point	
the mean is $\frac{1 \cdot 122 + 2 \cdot 127 + ... + 2 \cdot 157}{50} = 140.4$ nails.	2 points	
Total:	**4 points**	