ÉRETTSÉGI VIZSGA • 2021. október 19.

MATEMATIKA
ANGOL NYELVEN

KÖZÉPSZINTŰ
ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$A \cap B = \{3; 4; 5; 6\}$	2 points	
Total:	**2 points**	

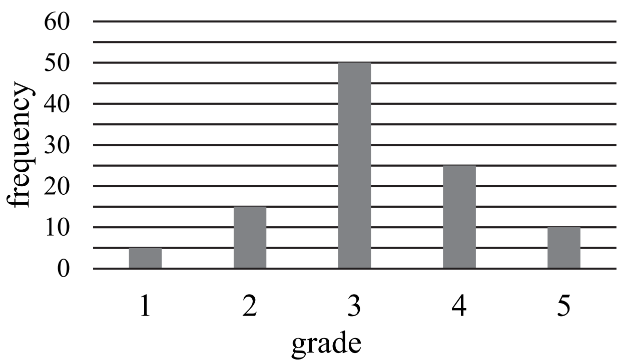
2.		
(6! =) 720	2 points	
Total:	**2 points**	

3.		
B, C, E	3 points	*Award 1 point for each correct answer and –1 point for each incorrect one. The total score must not be negative.*
Total:	**3 points**	

4.		
(10 200 : 0.85 =) 12 000 (Ft)	2 points	
Total:	**2 points**	

5.		
The minimum is at 3.	1 point	
The minimum value is –1.	1 point	
Total:	**2 points**	

6.		
($\sqrt[3]{729\,000}$ =) 90 (cm)	2 points	
Total:	**2 points**	

7.		
	3 points	*Award 1 point for the data shown on the horizontal axis, 1 point for the proportional scaling of the vertical axis and 1 point for the correct heights of the 5 bars.*
Total:	**3 points**	

8.		
$x = 5$	2 points	
Total:	**2 points**	

9.		
(100 = 16·6 + 4, that is) when 100 is divided by 6 the remainder is 4,	2 points	*A six-day cycle will end on the 96th day.*
and therefore the guard will work on that day.	1 point	
Total:	**3 points**	

10.		
The second term is (5·(–2) + 1 =) –9.	1 point	
The third term is ((–9)·(–2) + 1 =) 19.	1 point	
Total:	**2 points**	

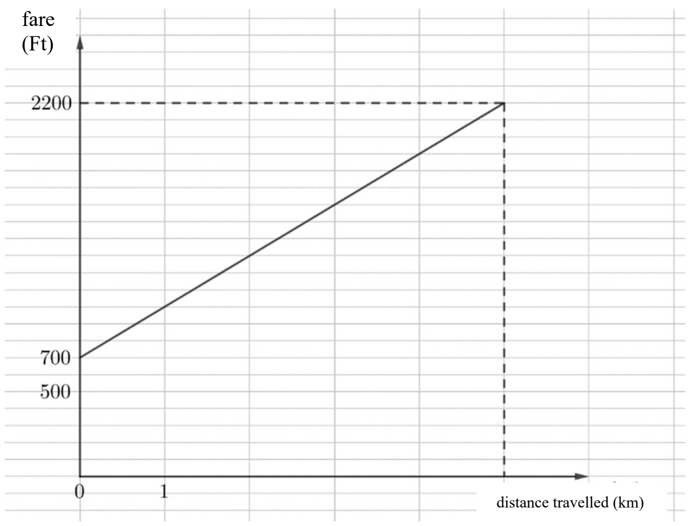
11.		
$r\,(=\sqrt{(-1-3)^2+(5-2)^2}\,)=5$	2 points	
$(x-3)^2+(y-2)^2=25$	2 points	
Total:	**4 points**	

12.		
When two dice are rolled the total number of possible outcomes is (6·6 =) 36.	1 point	
There will be three among these where the sum is 11 or more: 5-6, 6-5, 6-6.	1 point	
The probability is: $\frac{3}{36}(=0.08\dot{3})$.	1 point	
Total:	**3 points**	

II. A

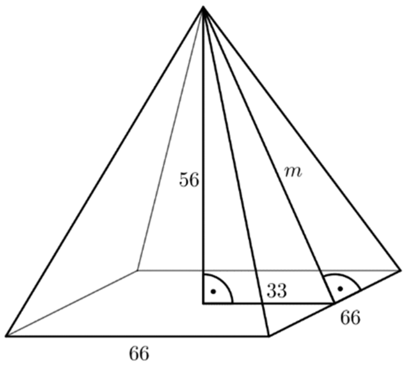
13. a)		
700 + 12.5 · 300 =	1 point	
= 4450 (Ft)	1 point	
Total:	**2 points**	

13. b)		
Let x be the distance travelled in km-s. In this case: 700 + x·300 = 2275,	1 point	
the distance travelled is therefore x = 5.25 (km).	1 point	
Total:	**2 points**	

13. c)		
		
The candidate draws a line segment,	1 point	
one endpoint of which is (0; 700),	1 point	
and the other is (5; 2200).	1 point	
Total:	**3 points**	

13. d) Solution 1		
Let *a* be the fixed fee (in forints) and let *k* be the fee for one kilometre. In this case $\left.\begin{aligned} a+6.5k&=2825 \\ a+10.4k&=4190 \end{aligned}\right\}.$	2 points	
$3.9k = 1365$	1 point	
The distance-fee is $k = 350$ (Ft),	1 point	
the fixed fee is $a = 550$ (Ft) (and these values are, indeed, solutions of the original problem).	1 point	
Total:	**5 points**	

13. d) Solution 2		
The second ride was (10.4 – 6.5 =) 3.9 kilometres longer than the first and it also cost (4190 – 2825 =) 1365 forints more.	3 points	
The distance fee is therefore 1365 : 3.9 = 350 (Ft).	1 point	
The fixed fee is $2825 - 6.5\cdot350 = 550$ (Ft).	1 point	$4190 - 10.4\cdot350 = 550$
Total:	**5 points**	

14. a)		
 The slant height m of the pyramid (according to the Pythagorean Theorem) is: $m=\sqrt{33^2+56^2}=65$ (cm).	2 points	
The area of one lateral face: $T=\frac{66\cdot 65}{2}$ (= 2145 cm^2).	1 point	
The total surface area: $A = 66^2 + 4\cdot 2145 = 12\ 936$ cm^2.	2 points	
Total:	**5 points**	

14. b) Solution 1		
The top edge of the truncated pyramid is half as long as the base edge, its length is 33 (cm).	1 point	*These 2 points are also due if the correct reasoning is reflected only by the solution.*
The height of the truncated pyramid is half of the height of the pyramid, it is 28 (cm).	1 point	
The volume of the truncated pyramid: $V=\frac{28}{3}\cdot(66^2+33^2+\sqrt{66^2\cdot 33^2})=71\ 148$ cm^3.	2 points	
Total:	**4 points**	

14. b) Solution 2		
The volume of the original pyramid is $\frac{66^2\cdot 56}{3}=81\ 312$ (cm^3).	1 point	
The smaller pyramid that is the top half after the cut is similar to the original pyramid, the ratio of the similarity being 1:2, and so the volume of the smaller pyramid would be $\left(\frac{1}{2}\right)^3=\frac{1}{8}$ of the original volume.	1 point	*The volume of the smaller pyramid:* $\frac{33^2\cdot 28}{3}=10\ 164$ (cm^3).
The volume of the truncated part is therefore $1-\frac{1}{8}=\frac{7}{8}$ of the original volume,	1 point	$V_{\substack{\text{truncated}\\\text{pyramid}}}=V_{\text{pyramid}}-V_{\substack{\text{smaller}\\\text{pyramid}}}$
which is $\frac{7}{8}\cdot 81\ 312=71\ 148$ cm^3.	1 point	81 312 – 10 164 = = 71 148 cm^3
Total:	**4 points**	

14. c)		
The sum of the degrees of all vertices in any graph is an even number.	1 point	*The number of vertices with an odd degree in any graph is even.*
(As $3 \cdot 7 = 21$ is odd) there is no such graph with 7 vertices.	1 point	*(As both 7 and 3 are odd numbers) such graph does not exist.*
Total:	**2 points**	

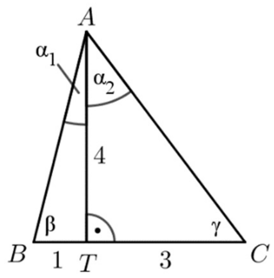
15. a)		
The median of Dávid's grades is 3,	1 point	
the mean is 3.8.	1 point	
The median of János' grades is 4, the mean is 2.8.	1 point	
When János' grades are arranged in increasing order, the third grade is a 4. As $2.8 \cdot 5 = 14$, the sum of the other four grades is 10. The fourth and fifth grades must be 4, too. (He must not have a 5, as the sum would then be greater than 14.) The sum of the remaining two grades may only be 2 if both of them are 1-s.	2 points	*A less detailed explanation may also be accepted.*
János' grades must be 1, 1, 4, 4, 4.	1 point	
Total:	**6 points**	

15. b)		
The sum of the grades for the whole year is $9 \cdot 3 + 6 \cdot 4.5 = 54$.	1 point	
As there were a total 15 grades the mean is $\frac{54}{15} =$	1 point	
$= 3.6$.	1 point	
Total:	**3 points**	

15. c)		
There are $\binom{5}{2} = 10$ ways to select two numbers out of five. (Total number of cases.)	1 point	1-2, 1-3, 1-4, 1-5, 2-3, 2-4, 2-5, 3-4, 3-5, 4-5
The mean of the selected numbers will be an integer if and only if their sum is even, i.e. in case of two even or two odd numbers.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Favourable cases are: 1-3, 1-5, 3-5 and 2-4, four cases altogether.	1 point	
The probability is: $\frac{4}{10} = 0.4$.	1 point	
Total:	**4 points**	

II. B

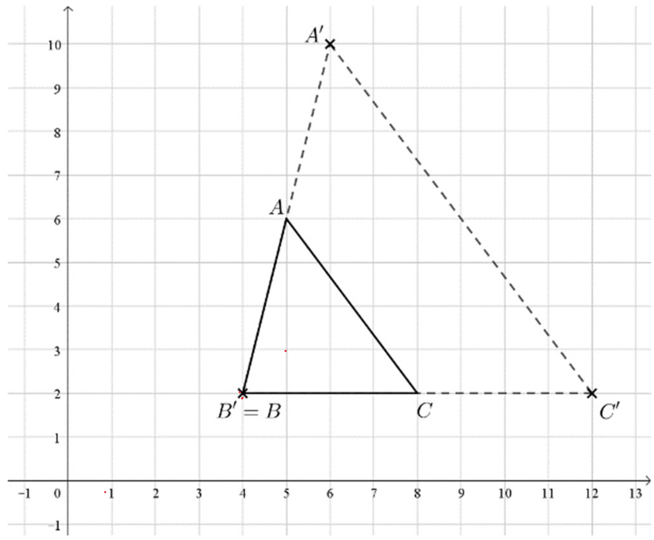
16. a) **Solution 1**		
The length of side *AB* of the triangle (e.g. applying the distance formula) is $AB=\sqrt{1^2+4^2}=\sqrt{17}$ (≈ 4.12).	1 point	
Similarly, $AC=\sqrt{3^2+4^2}=5$.	1 point	
Let α be the angle we are looking for. Apply the cosine rule ($BC = 4$): $16=17+25-2\cdot\sqrt{17}\cdot5\cdot\cos\alpha$.	2 points	*Calculating the area of triangle ABC in two different ways:* $\frac{4\cdot4}{2}=\frac{5\cdot\sqrt{17}\cdot\sin\alpha}{2}$.
$\cos\alpha\approx0.6306$,	1 point	$\sin\alpha\approx0.7761$ *(where α is acute)*
so $\alpha\approx50.91°$.	1 point	
Total:	**6 points**	

16. a) **Solution 2**		
(Use the symbols of the diagram, the angle is $\alpha=\alpha_1+\alpha_2$.) $BT = 1$, $TC = 3$, $AT = 4$ 	1 point	
In triangle *ABT*: $\tan\alpha_1=\frac{1}{4}$,	1 point	$\tan\beta=4$
$\alpha_1\approx14.04°$.	1 point	$\beta\approx75.96°$
In triangle *ATC* $\tan\alpha_2=\frac{3}{4}$,	1 point	$\tan\gamma=\frac{4}{3}$
$\alpha_2\approx36.87°$.	1 point	$\gamma\approx53.13°$
So $\alpha=\alpha_1+\alpha_2=50.91°$.	1 point	$\alpha=180°-\beta-\gamma=$ $=50.91°$
Total:	**6 points**	

16. a) Solution 3		
$\overrightarrow{AB}=(-1;-4)$, $\overrightarrow{AC}=(3;-4)$.	1 point	
The lengths of the vectors: $\lvert\overrightarrow{AB}\rvert=\sqrt{17}$, $\lvert\overrightarrow{AC}\rvert=5$.	1 point	
The angle is the one between the two vectors, let it be α. Give the scalar product of the two vectors in two different ways: $\sqrt{17}\cdot 5\cdot\cos\alpha=(-1)\cdot 3+(-4)\cdot(-4)$.	2 points	
cos α ≈ 0.6306,	1 point	
so α ≈ 50.91°.	1 point	
Total:	**6 points**	

16. b)		
This altitude passes through point *B* and one possible normal vector is $\overrightarrow{AC}$ (3; –4).	2 points	
The equation of the altitude is $3x-4y=3\cdot 4+(-4)\cdot 2$ or $3x-4y=4$.	2 points	
The orthocentre (also being on the altitude through *A* whose equation is $x = 5$) has an *x* coordinate of 5,	1 point	
while the *y* coordinate is obtained by substituting $x=5$ into the equation of the altitude through *B*: $y = 2.75$. (I.e. *M*(5; 2.75))	2 points	
Total:	**7 points**	

Note: The equation of the altitude through vertex C is $x + 4y = 16$.

16. c)		
	1 point	*This point is also due if the candidate gives the correct answer without a diagram.*
The image of point *B* is itself (i.e. *B'*(4; 2)).	1 point	
C'(12; 2)	1 point	
A'(6; 10)	1 point	
Total:	**4 points**	

17. a)		
The common difference of the sequence is $d=\frac{81-24}{3}=19$,	2 points	
The first term is (24 – 19 =) 5.	1 point	
The sum of the first 16 terms is $\frac{2\cdot5+15\cdot19}{2}\cdot16=2360$.	2 points	
The 106th term of the sequence is $5+105\cdot19=2000$.	1 point	
$\frac{2360}{2000}=1.18$,	1 point	
so, the sum of the first 16 terms is 18% greater than the 106th term of the sequence.	1 point	
Total:	**8 points**	

17. b)		
The common ratio of the sequence is $q=\sqrt[3]{\frac{81}{24}}=1.5$,	2 points	
The first term is (24 : 1.5 =) 16.	1 point	
(The n^{th} term of the sequence is $a_n=16\cdot1.5^{n-1}$.) The equation $16\cdot1.5^{n-1}=10\ 000\ 000$ is to be solved.	2 points	
$1.5^{n-1}=625\ 000$	1 point	
$n-1=\log_{1.5}625000$	1 point	$n-1=\frac{\log625000}{\log1.5}$
$n\approx33.9$	1 point	
(As the sequence is strictly monotone increasing) 33 terms of the sequence is less than 10 000 000.	1 point	
Total:	**9 points**	

Notes:
1. Award full score if the candidate, after obtaining the first term and the common ratio, correctly guesses (through trial and error) that the 33rd term of the sequence is still less than 10 000 000, while the 34th term is greater.
2. The appropriate points are also due if the candidate correctly solves an inequality, rather than an equation.

18. a) Solution 1		
Let x be the number of students attending the high level physics course. The number of math students is then $2x$. As per the original problem: $2x+x-6=15$.	2 points	
Here $x=7$,	1 point	*There are* $2\cdot7=14$ *math students.*
and there are (15 – 7 =) 8 students in the class who only attend the high level mathematics course.	1 point	$14-6=8$.
Total:	**4 points**	

18. a) **Solution 2**		
Let a be the number of students attending the physics course only, and let b be the number of students attending the mathematics courses only. In this case $\left.\begin{array}{r} a+b+6=15 \\ 2\cdot(a+6)=b+6 \end{array}\right\}$.	2 points	
From the first equation $a = 9 - b$.	1 point	
Substituting into the second equation it turns out there are $b = 8$ students in the class attending high level mathematics only.	1 point	
Total:	**4 points**	

18. b) **Solution 1**		
Let the sides of the screen be $16a$ and $9a$.	1 point	
The length of the horizontal side of a small rectangle is $\frac{16a}{6}$,	1 point	
The length of the vertical side is $\frac{9a}{4}$.	1 point	
The ratio of these two is: $\frac{16a}{6} : \frac{9a}{4} = \frac{16a}{6} \cdot \frac{4}{9a} =$	1 point	
$= \frac{64}{54} \left(= \frac{32}{27} \right)$.	1 point	
Total:	**5 points**	

18. b) **Solution 2**		
Let x be the length of the horizontal side of a small rectangle and let y be the length of the vertical side. The horizontal side of the screen is then $6x$, while the vertical side is $4y$.	2 points	
As $\frac{6x}{4y} = \frac{16}{9}$,	1 point	
$\frac{x}{y} = \frac{16}{9} \cdot \frac{4}{6} =$	1 point	
$= \frac{64}{54} \left(= \frac{32}{27} \right)$.	1 point	
Total:	**5 points**	

18. c) **Solution 1**		
There are 24 slots to place Stefi's rectangle. After that, there are 23 slots for Cili's. This gives an overall $24\cdot 23 = 552$ possible arrangements.	2 points	*(If the two rectangles are treated as equals) the total number of possible cases is* $\binom{24}{2} = 276$.
The number of favourable cases is $6\cdot 5 = 30$.	2 points	*There are* $\binom{6}{2} = 15$ *different ways to select 2 slots from the 6 in the top row.*
The probability is $\frac{30}{552}$ (≈ 0.054).	1 point	$\frac{15}{276}$
Total:	**5 points**	

18. c) **Solution 2**		
(Let us place both rectangles somewhere on the screen.) The probability that Stefi's rectangle will be in the top row is $\frac{6}{24}$.	1 point	
After this, the probability that Cili's rectangle will also be in the top row is $\frac{5}{23}$.	2 points	
The overall probability is the product of these two: $\frac{30}{552}$ (≈ 0.054).	2 points	
Total:	**5 points**	

18. d)		
The product 24! contains the factors 5, 10, 15 and 20,	1 point	
as well as (for example) the number 2.	1 point	$2\cdot 5\cdot 10\cdot 15\cdot 20 = 30\,000$
The product of these numbers, and therefore 24! itself, is divisible by 10 000.	1 point	
Total:	**3 points**	