ÉRETTSÉGI VIZSGA • 2022. május 3.

MATEMATIKA
ANGOL NYELVEN

KÖZÉPSZINTŰ
ÍRÁSBELI VIZSGA

minden vizsgázó számára

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

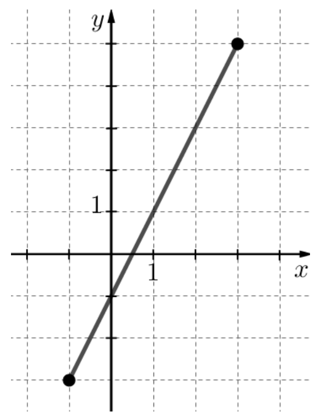
I.

1.		
$B = \{1; 2; 3; 4\}$	2 points	
Total:	**2 points**	

2.		
(With the Pythagorean theorem: $\sqrt{26^2 - 10^2} =$) 24 cm	2 points	
Total:	**2 points**	

3.		
-3	2 points	
Total:	**2 points**	

4.		
$(840 : 0.35 =)$ 2400 Ft	2 points	
Total:	**2 points**	

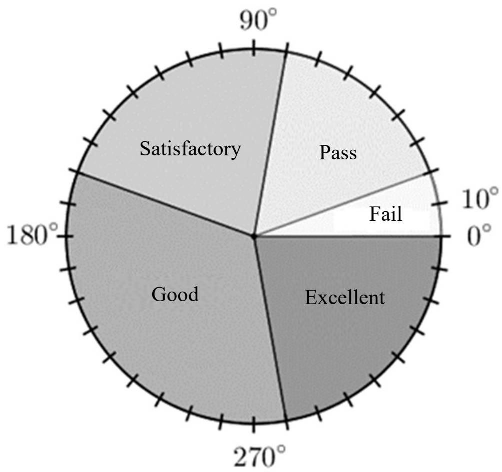
5.		
The graph is that of a linear function whose gradient is 2,	1 point	
the y-intercept is at -1,	1 point	
and is restricted to the correct interval.	1 point	
Total:	**3 points**	

6.		
The mean of the five numbers given is 4,	1 point	*Some examples of a correct solution: 3, 3, 3, 3, 8 or 2, 3, 3, 3, 9 or 1, 2, 3, 3, 11.*
the single mode is 3.	1 point	
Total:	**2 points**	

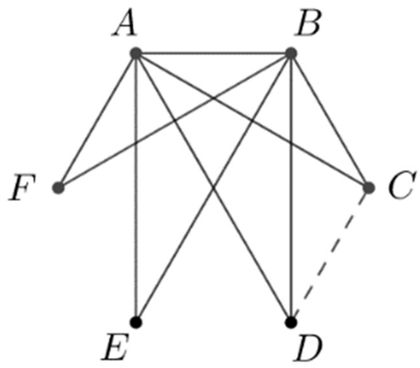
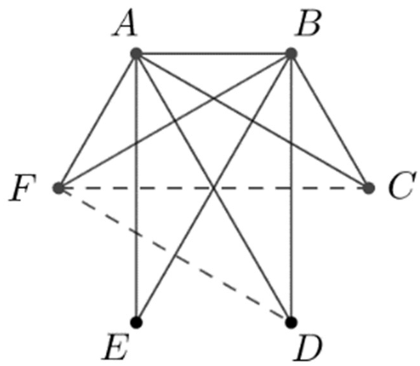
7.		
g, *i*	2 points	*Award 1 point for a single correct answer or two correct and one incorrect answers.*
Total:	**2 points**	

8.		
$\left(\frac{(10-2)\cdot 180°}{10}=180°-\frac{360°}{10}=\right)144°$	2 points	
Total:	**2 points**	

9.		
$4^x = 32$	1 point	
$x = \log_4 32$	1 point	$2^{2x} = 2^5$
$x = 2.5$	1 point	
Total:	**3 points**	

10.		
360° : 18 = 20°, so the central angles of the grades are 20°, 60°, 80°, 120°, 80°, respectively.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
	2 points	
Total:	**3 points**	

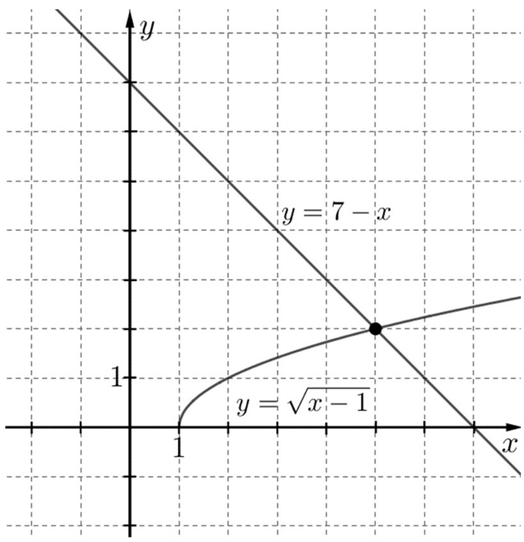
11.		
1122, 1212, 2112	3 points	*Each correct number is worth 1 point. Deduce a total of 1 point if the candidate also lists incorrect solution(s).*
Total:	**3 points**	

12.		
She might have 2 acquaintances,	1 point	
the corresponding graph: 	1 point	.
She might have 4 acquaintances,	1 point	
the corresponding graph: 	1 point	
Total:	**4 points**	

II. A

13. a)		
Use a common denominator: $\frac{9x+3}{6}+\frac{2x-2}{6}=13$	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$9x + 3 + 2x - 2 = 78$	1 point	
$11x = 77$	1 point	
$x = 7$	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**5 points**	

13. b) Solution 1		
Square: $x-1=49-14x+x^2$.	1 point	
$x^2-15x+50=0$	1 point	
The solutions are $x_1 = 5$ and $x_2 = 10$.	2 points	
Check: 5 is a correct solution of the original equation,	1 point	*These 2 points are also due if the candidate refers to equivalent steps while stating* $1 \le x \le 7$.
10 is incorrect.	1 point	
Total:	**6 points**	

<table>
<tr><td colspan="3">13. b) Solution 2</td></tr>
<tr><td>Graphical solution:

</td><td>4 points</td><td>2 points for correctly graphing the square root function,
2 points for correctly graphing the linear function.</td></tr>
<tr><td>As seen in the graph: $x = 5$.</td><td>1 point</td><td></td></tr>
<tr><td>Check by substitution.</td><td>1 point</td><td></td></tr>
<tr><td>Total:</td><td>6 points</td><td></td></tr>
</table>

14. a)		
(Let q be the common ratio of the geometric sequence:) $a_4 = 0.75 \cdot q^3 = 6$.	1 point	
$q^3 = 8$,	1 point	
$q = 2$.	1 point	
$S_{20} = 0.75 \cdot \frac{2^{20} - 1}{2 - 1} =$	1 point	
$= 786\,431.25$	1 point	
Total:	**5 points**	

14. b) Solution 1		
Let a be the first term of the sequence and let d be the common difference. In this case: $a + (a + d) + (a + 2d) = 18$,	1 point	
also $(a + 2d) + (a + 3d) = a + (a + d) + 28$.	1 point	*The 3rd term is 2d more than the 1st, the 4th term is also 2d more than the 2nd, so 4d = 28.*
From the second equation $d = 7$.	2 points	
Substitute into the first equation: $a = –1$.	1 point	
The 20th term of the sequence is $a_{20} = 132$. $S_{20} = \frac{-1+132}{2} \cdot 20 = 1310$	2 points	$-1 + 6 + 13 + \ldots + 132 = 1310$
Total:	**7 points**	

14. b) Solution 2		
Let b be the second term of the sequence and let d be the common difference. In this case: $(b – d) + b + (b + d) = 18$,	1 point	
from this $b = 6$.	1 point	
As per the second condition: $(b + d) + (b + 2d) = (b – d) + b + 28$,	1 point	
$d = 7$.	1 point	
The first term of the sequence is $b – d = –1$.	1 point	
$S_{20} = \frac{2 \cdot (-1) + 19 \cdot 7}{2} \cdot 20 = 1310$	2 points	
Total:	**7 points**	

15. a)		
The volume of the box: $V = 13^2 \cdot \pi \cdot 18 \approx$	1 point	
≈ 9557 cm^3,	1 point	
i.e. about 9.6 litres.	2 points	*Award 1 point for the correct exchange of units, and 1 point for correct rounding.*
Total:	**4 points**	

15. b)		
The surface area of such a cylinder is $2 \cdot 13^2 \cdot \pi + 2 \cdot 13 \cdot \pi \cdot 18 \approx$	1 point	
≈ 2532 cm^2.	1 point	
The area of the metal sheet used to make one such box is $1.18 \cdot 2532 \approx 2988$ cm$^2 \approx$	1 point	
≈ 0.3 m^2,	1 point	
The total area needed for 1000 boxes is about 300 m^2.	1 point	
Total:	**5 points**	

15. c) Solution 1		
As 800 : 2000 = 2 : 5, let the price (in forints) of the smallest box be $2x$, and let the price of the middle-sized box be $5x$.	1 point	
$2x + 5x = 2100$	1 point	
$x = 300$	1 point	
The price of the smallest box is ($2 \cdot 300 =$) 600 Ft, the price of the middle-sized box is ($5 \cdot 300 =$) 1500 Ft.	1 point	
Total:	**4 points**	

15. c) Solution 2		
Let the price (in forints) of the smallest box be $800x$, the price of the middle-sized box is $2000x$.	1 point	*A total of 2800 cm^2 of metal sheet costs a total of 2100 Ft. The cost of 1 cm^2 is therefore 2100 : 2800 = 0.75 Ft.*
$800x + 2000x = 2100$	1 point	
$x = 0.75$	1 point	
The price of the smallest box is ($800 \cdot 0.75 =$) 600 Ft, the price of the middle-sized box is ($2000 \cdot 0.75 =$) 1500 Ft.	1 point	
Total:	**4 points**	

II. B

16. a) **Solution 1**		
One direction vector of the line is: $\overrightarrow{AB}(1; -4)$,	1 point	
one normal vector: (4; 1),	1 point	$-4x - y = -4$
the equation of the line is: $4x + y = 4$.	1 point	
Total:	**3 points**	

16. a) **Solution 2**		
The line intersects the y-axis at 4,	1 point	
the gradient is $m = -4$,	1 point	
and so the equation is $y = -4x + 4$.	1 point	
Total:	**3 points**	

16. b) **Solution 1**		
$\overrightarrow{AD} = \big((5;6) - (0;4) =\big)(5;2)$ $\overrightarrow{BC} = \big((6;2) - (1;0) =\big)(5;2)$	2 points	$\overrightarrow{AB} = \overrightarrow{DC} = (1; -4)$
As two opposite side-vectors of the quadrilateral are equal (and so it does have a pair of parallel and equal opposite sides) the statement is proven.	1 point	
Total:	**3 points**	

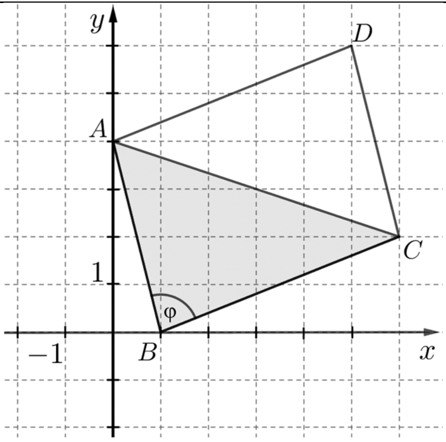
16. b) **Solution 2**		
The length of each side of the quadrilateral: $\lvert AB\rvert = \lvert DC\rvert = \sqrt{1^2 + (-4)^2} = \sqrt{17}$, $\lvert AD\rvert = \lvert BC\rvert = \sqrt{5^2 + 2^2} = \sqrt{29}$.	2 points	
As the opposite sides of the quadrilateral are equal the statement is proven.	1 point	
Total:	**3 points**	

16. b) **Solution 3**		
The gradient of both sides AB and DC is -4 (e.g. based on a diagram). The gradient of both sides AD and BC is $\frac{2}{5}$.	2 points	
As the gradients of the opposite sides are equal, those are parallel, and so the statement is proven.	1 point	
Total:	**3 points**	

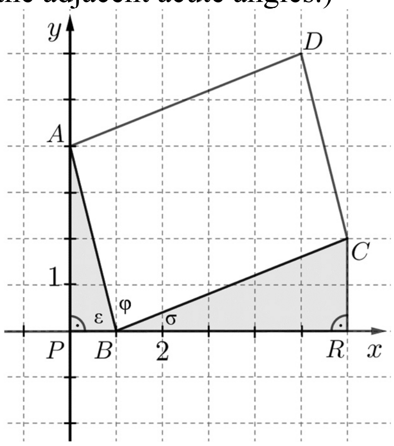
16. b) Solution 4

The midpoints of the diagonals of the quadrilateral are $F_{AC} = \left(\left(\frac{0+6}{2}; \frac{4+2}{2}\right) = \right)(3; 3)$, $F_{BD} = \left(\left(\frac{1+5}{2}; \frac{0+6}{2}\right) = \right)(3; 3)$.	2 points	
As the two midpoints coincide (the diagonals bisect each other) the statement is proven.	1 point	*The quadrilateral has central symmetry, so the statement is proven.*
Total:	**3 points**	

16. c) Solution 1

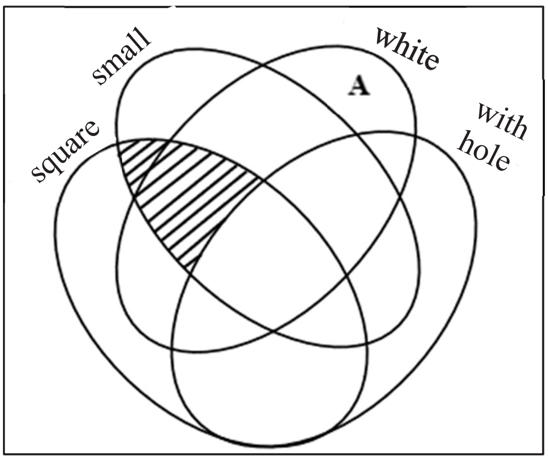
 The lengths of the sides of the angle are: $AB = \sqrt{4^2+1^2} = \sqrt{17}$, $BC = \sqrt{5^2+2^2} = \sqrt{29}$.	1 point	
The length of diagonal AC is: $\sqrt{6^2+2^2} = \sqrt{40}$.	1 point	
Let φ denote the particular angle of triangle *ABC*. Use the Law of Cosines: $\sqrt{40}^2 = \sqrt{17}^2 + \sqrt{29}^2 - 2 \cdot \sqrt{17} \cdot \sqrt{29} \cdot \cos\varphi$.	1 point	
$\cos\varphi = \frac{3}{\sqrt{17} \cdot \sqrt{29}}$ (≈ 0.1351),	2 points	
$\varphi \approx 82.2°$.	1 point	
Total:	**6 points**	

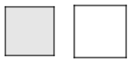
16. c) **Solution 2**		
The coordinates of the two vectors enclosing the angle: $\overrightarrow{BA}=(-1;4)$; $\overrightarrow{BC}=(5;2)$.	1 point	
The scalar product with coordinates: $\overrightarrow{BA}\cdot\overrightarrow{BC}=(-1)\cdot5+4\cdot2=3$.	1 point	
The lengths of the vectors: $\lvert\overrightarrow{BA}\rvert=\sqrt{(-1)^2+4^2}=\sqrt{17}$, $\lvert\overrightarrow{BC}\rvert=\sqrt{5^2+2^2}=\sqrt{29}$.	1 point	
Let φ denote the angle enclosed, use the definition of the scalar product: $\overrightarrow{BA}\cdot\overrightarrow{BC}=\sqrt{17}\cdot\sqrt{29}\cdot\cos\varphi$.	1 point	
As the above scalar products are equal: $\cos\varphi=\frac{3}{\sqrt{17}\cdot\sqrt{29}}$,	1 point	
azaz $\varphi\approx82.2°$.	1 point	
Total:	**6 points**	

16. c) **Solution 3**		
(Let $P(0;0)$ be the perpendicular projection of point A on the x-axis and let $R(6;0)$ be the perpendicular projection of point C. Let φ, denote the angle, and let ε and σ be the adjacent acute angles.) 	1 point	
In the right triangle APB: $\tan\varepsilon=\frac{4}{1}$,	1 point	
$\varepsilon\approx76.0°$.	1 point	
In the right triangle BRC: $\tan\sigma=\frac{2}{5}$,	1 point	
$\sigma\approx21.8°$.	1 point	
$\varphi=180°-\sigma-\varepsilon\approx82.2°$	1 point	
Total:	**6 points**	

16. d) **Solution 1**		
The four vertices can be labelled by the letters *E, F, G* and *H* in $(4\cdot3\cdot2\cdot1=)$ 24 different ways (total number of cases).	1 point	
Vertex *E* can be at four different positions.	1 point	*Let E and F be the end-points of one side. This gives* $4\cdot2=8$ *possibilities.*
The lettering may continue in two directions (once the direction is set, the order is also determined).	1 point	*In both cases vertices G and H may only be placed in one position correctly.*
The number of favourable cases is therefore $4\cdot2=8$.	1 point	*The number of favourable cases is therefore 8.*
The probability: $\frac{8}{24}\left(=\frac{1}{3}\right)$.	1 point	
Total:	**5 points**	

16. d) **Solution 2**		
Let us fix the position of letter *E* in one of the vertices.	1 point	
The other letters may be placed in 3! different ways, the total number of cases is therefore 6.	1 point	
Two of these are favourable, i.e. lettered correctly.	2 points	
The probability: $\frac{2}{6}\left(=\frac{1}{3}\right)$.	1 point	
Total:	**5 points**	

17. a)		
	2 points	
Total:	**2 points**	

17. b)		
Circle the small, grey square without hole and the small white square without hole. 	2 points	*Award 1 point for a single correct answer or two correct and one incorrect answers. Award 0 points in any other case.*
Total:	**2 points**	

17. c) Solution 1		
The total number of possibilities is $\binom{16}{2}$.	1 point	
There are 4 small triangles in the set. There are $\binom{4}{2}$ ways to select two of them (number of favourable cases).	1 point	
The probability: $\dfrac{\binom{4}{2}}{\binom{16}{2}} =$	1 point	
$= \dfrac{6}{120}$ (= 0.05).	1 point	
Total:	**4 points**	

17. c) Solution 2		
As there are 4 small triangles in the set, the probability that the first piece will be a small triangle is: $\dfrac{4}{16}$.	1 point	
The probability that the second piece is also a small triangle is: $\dfrac{3}{15}$.	1 point	
The final probability: $\dfrac{4}{16} \cdot \dfrac{3}{15} =$	1 point	
$= \dfrac{12}{240}$ (= 0.05).	1 point	
Total:	**4 points**	

17. d) **Solution 1**		
The length of one side of the triangle: $AC = 3\sqrt{2}$ (≈ 4.24) (cm),	2 points	$AC=\sqrt{3^2+3^2}=\sqrt{18}$
the length of the other side: CE = 3 (cm),	1 point	
the angle between these sides: $ACE\angle = 45° + 60° = 105°$.	1 point	
$A_{ACE} = \dfrac{3\sqrt{2}\cdot 3\cdot \sin 105°}{2} \approx$	1 point	
≈ 6.15 (cm^2)	1 point	
Total:	**6 points**	

17. d) **Solution 2**		
The combined area of the square and the regular triangle is $3^2+\dfrac{3^2\cdot\sqrt{3}}{4}=9+2.25\cdot\sqrt{3}$ (≈ 12.90 cm^2).	2 points	
The height that belongs to the side AB in triangle ABE is 1.5 cm, therefore the area of triangle ABE is 2.25 (cm^2).	1 point	
The area of triangle ADC is 4.5 (cm^2).	1 point	
The final area is: $9+2.25\cdot\sqrt{3}-2.25-4.5\approx$	1 point	
≈ 6.15 (cm^2).	1 point	
Total:	**6 points**	

17. e) **Solution 1**		
BA = 3 cm, BC = 3 cm, BE = 3 cm	1 point	
As point B is equidistant from all three of the above points it must be the centre of the circumcircle of triangle ACE. (The statement is therefore proven.)	2 points	
Total:	**3 points**	

17. e) **Solution 2**		
The centre of the circumcircle of a triangle is the point of intersection of any two perpendicular bisectors.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The perpendicular bisector of AC (diagonal) is the line of diagonal BD which passes through point B.	1 point	
The perpendicular bisector of side EC (the altitude of the regular triangle BCE) also passes through point B. (The statement is therefore proven.)	1 point	
Total:	**3 points**	

18. a)		
(As both numbers were even with their sum being less than 6) the numbers rolled must have been 2, 2.	1 point	
Andrea gained ($4\cdot20 + 3\cdot20 + 2\cdot20 =$) 180 points.	1 point	
Andrea's final score increased by ($180 - 60 =$) 120 points,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
being ($120 + 120 =$) 240 at the end of round 2.	1 point	
Total:	**4 points**	

18. b)		
Two odd numbers were rolled, their sum being at least 6	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
so the possible pairs are 1, 5 (in any order) 3, 3 3, 5 (in any order) 5, 5.	2 points	
Total:	**3 points**	

18. c)		
He placed x forints on event A and therefore $2x$ on event E and $70 - 3x$ on event D.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
In this case: $4\cdot x + 2\cdot(70 - 3x) + 3\cdot 2x = 200$.	2 points	
$4x + 140 = 200$	1 point	
$x = 15$, so Balázs placed 15 points on event A.	1 point	
Check: (30 points on event E, $75 - 45 = 25$ points on event D, his gain is $4\cdot15 + 2\cdot25 + 3\cdot30 = 200$).	1 point	
Total:	**6 points**	

18. d) Solution 1		
There are $(6 \cdot 6 \cdot 6 =)$ 216 different ways to roll three dice.	1 point	
$(5 \cdot 5 \cdot 5 =)$ 125 of these will have no 5-s.	1 point	
$(216 - 125 =)$ 91 cases will have at least one 5.	1 point	
The probability: $\frac{91}{216}$ (≈ 0.42).	1 point	
Total:	**4 points**	

18. d) Solution 2		
The probability of rolling something other than 5 on one die is: $\frac{5}{6}$.	1 point	
The probability of rolling three dice, none of them showing 5 is: $\left(\frac{5}{6}\right)^3$.	1 point	
The probability that at least one of the dice will show 5 is: $1-\left(\frac{5}{6}\right)^3 =$	1 point	
$=\frac{91}{216}$.	1 point	
Total:	**4 points**	

18. d) Solution 3		
The probability of rolling three 5-s is: $\left(\frac{1}{6}\right)^3 = \frac{1}{216}$.	1 point	$\frac{1}{6^3}$
The probability of rolling exactly two 5-s with three dice is: $\binom{3}{2} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right) = \frac{15}{216}$.	1 point	$\frac{\binom{3}{1} \cdot 1^2 \cdot 5}{6^3}$
The probability of rolling exactly one 5 with three dice is: $\binom{3}{1} \cdot \left(\frac{1}{6}\right) \cdot \left(\frac{5}{6}\right)^2 = \frac{75}{216}$.	1 point	$\frac{\binom{3}{2} \cdot 1 \cdot 5^2}{6^3}$
The final probability is the sum of the above: $\frac{91}{216}$.	1 point	
Total:	**4 points**	