ÉRETTSÉGI VIZSGA • 2024. május 7.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without that remark as well.

6. Deduct points for **missing units** only if the missing unit is part of the answer or a unit exchange (without parentheses).
7. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
1; 2; 4; 7; 14; 28	2 points	*Award 1 point in case of 6 correct and one incorrect divisor, also in case of 4 or 5 correct and 0 incorrect divisor. Award 0 points in any other case.*
Total:	**2 points**	

2.		
$(6 \cdot 180° =)\ 1080°$	2 points	
Total:	**2 points**	

3.		
($8 = 2^3$, so it takes $3 \cdot 10 =$) 30 days.	2 points	
Total:	**2 points**	

4.		
The mean is $\left(\frac{6 \cdot 41 + 7 \cdot 42 + 9 \cdot 43 + 3 \cdot 44}{25} =\right) 42.36.$	2 points	
The mode is 43.	1 point	
The median is 42.	1 point	
Total:	**4 points**	

5.		
$\left(\binom{16}{2} =\right) 120$	2 points	
Total:	**2 points**	

6.		
$\overrightarrow{CB} = \mathbf{p} - \mathbf{q}$	1 point	
$\overrightarrow{CF} = \frac{1}{2}(\mathbf{p} - \mathbf{q})$	1 point	
$\overrightarrow{BA} = -\mathbf{p}$	1 point	
Total:	**3 points**	

7.		
The total mass of the pills is 166 – 24.7 = 141.3 (g),	1 point	
so there are 141.3 : 1.57 =	1 point	
= 90 pills in a full box.	1 point	
Total:	**3 points**	

8.		
The converse of the statement: *If the sum of two numbers is even, then their product is odd.*	1 point	
The converse is false.	1 point	
Total:	**2 points**	

9.		
($1.06^3 \approx 1.191$, i.e.) the value increases by about 19.1%.	2 points	
Total:	**2 points**	

10.		
($2 = \frac{2}{3}x - 2$, i.e.) $x = 6$, the first coordinate of point P is 6.	2 points	
Total:	**2 points**	

11.		
$f(3) = 4$	2 points	
Total:	**2 points**	

12.		
There are a total ($9 \cdot 10 =$) 90 two-digit, positive integers (total number of cases).	2 points	
Of these, 9 are divisible by 11 (number of favourable cases).	1 point	
The probability: $\frac{9}{90} = 0.1$.	1 point	
Total:	**4 points**	

II. A

13. a) Solution 1		
Let k be the price of 1 kg of potatoes and let h be the price of 1 kg of onions. In this case: $\left.\begin{aligned} 4k+3h&=1570 \\ 2k+h&=700 \end{aligned}\right\}$.	2 points	
From the second equation $h = 700 - 2k$. Substitute it into the first equation: $4k + 2100 - 6k = 1570$.	1 point	*Using elimination:* $\left.\begin{aligned} 4k+3h&=1570 \\ 4k+2h&=1400 \end{aligned}\right\}$.
So, $k = 265$ Ft for 1 kg of potatoes,	1 point	*By subtracting the equations: $h = 170$,*
and $h = 700 - 2 \cdot 265 = 170$ Ft for 1 kg of onions.	1 point	*and $k = 265$.*

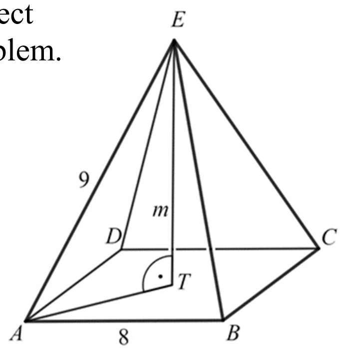
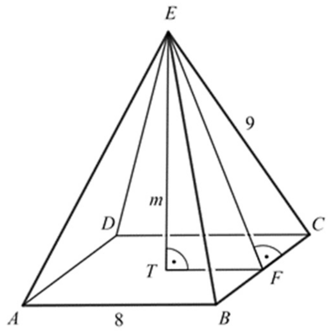
Check against the text of the problem: $4 \cdot 265 + 3 \cdot 170 = 1570$ Ft and $2 \cdot 265 + 170 = 700$ Ft, indeed.	1 point	
Total:	**6 points**	

13. a) Solution 2		
If 2 kg of potatoes and 1 kg of onions cost 700 Ft, then 4 kg of potatoes and 2 kg of onions cost 1400 Ft.	2 points	
It follows then, that 1 kg of onions is (1570 – 1400 =) 170 Ft.	2 points	
1 kg of potatoes then costs $(700 - 170) : 2 = 265$ Ft.	2 points	
Total:	**6 points**	

13. b)		
Do the distributions: $4 + 2x^2 - 2x = x^2 + 2x + 1$.	2 points	
Rearranged: $x^2 - 4x + 3 = 0$.	1 point	
$x_1 = 1$, $x_2 = 3$	2 points	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**6 points**	

14. a) Solution 1		
The base radius of Dóri’s cylinder is 3 cm.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The volume is $V = 3^2 \cdot \pi \cdot 10 = 90\pi$ (≈ 282.7) (cm^3).	1 point	
The volume of Panni’s 40 cm long cylinder of radius r is the same, so $r^2\pi \cdot 40 = 90\pi$,	1 point	
$r^2 = 2.25$.	1 point	
(as $r > 0$) $r = 1.5$ (cm).	1 point	
The diameter is, therefore, 3 cm.	1 point	
Total:	**6 points**	

14. a) Solution 2		
The volumes of the two cylinders are equal, but the length of Panni’s cylinder is 4 times that of Dóri’s.	1 point	
So, the base area of Panni’s cylinder must be one quarter of the base area of Dóri’s cylinder.	2 points	
(Any two circles are similar, so) if the ratio of the areas of two circles is $1 : 4$ then the ratio of their radii (also, the diameters) is the square root of that, $1 : 2$.	2 points	
Therefore, the diameter of Panni’s cylinder must be half of the diameter of Dóri’s cylinder, that is, 3 cm.	1 point	
Total:	**6 points**	

14. b)		
Diagram, reflecting correct interpretation of the problem. 	1 point	
The area of the base of the pyramid: $8^2 = 64$ (cm^2).	1 point	
Segment AT is half the diagonal of the square, so $AT = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$ (≈ 5.66) (cm).	1 point	*Use the Pythagorean Theorem in triangle EFC:* $EF^2 + 4^2 = 9^2$, $EF = \sqrt{65}$ (≈ 8.06) (cm).
Use the Pythagorean Theorem in the right triangle ATE: $(4\sqrt{2})^2 + m^2 = 9^2$,	1 point	*In the right triangle ETF:* $m^2 + 4^2 = (\sqrt{65})^2$,
then $m = 7$ (cm).	1 point	
The volume of the pyramid: $V = \frac{64 \cdot 7}{3} \approx 149.3$ cm^3.	1 point	
Total:	**6 points**	

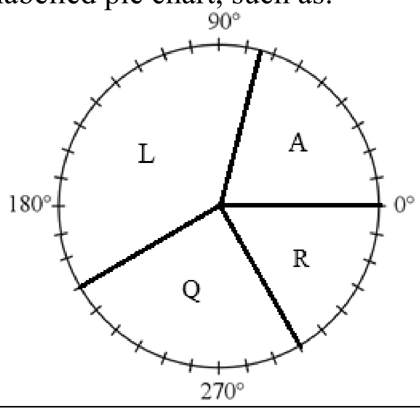
15. a)		
The graph has ($4 + 3 \cdot 2 =$) 10 edges	1 point	
and 11 vertices.	1 point	
Total:	**2 points**	

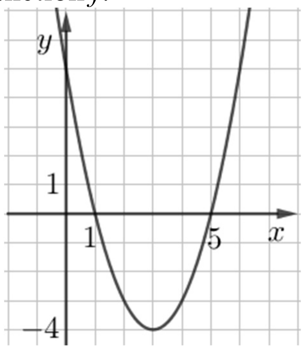
15. b)		
The number of new edges doubles every year so, in the second year, there will be $6 \cdot 2 = 12$ new edges.	2 points	*The number of new edges of the graph in each new year form consecutive terms of a geometric sequence with* $a_1 = 6$ *and* $q = 2$.
There are 24 new edges in the third year, and 48 new edges in the fourth.	1 point	$S_4 = 6 \cdot \frac{2^4 - 1}{2 - 1} =$
There are a total $6 + 12 + 24 + 48 = 90$ new edges.	1 point	$= 90$
Together with the original 4 edges, there will be $4 + 90 = 94$ edges in the graph by the end of the fourth year.	1 point	
Total:	**5 points**	

15. c)		
The number of saplings planted in each row form consecutive terms of an arithmetic sequence. The first term of the sequence is 12, the common difference is 3.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
In the last row: $a_{17} = a_1 + 16d = 12 + 16 \cdot 3 =$	1 point	
= 60 saplings.	1 point	
In the whole garden, there will be a total $S_{17} = \frac{a_1 + a_{17}}{2} \cdot 17 = \frac{12+60}{2} \cdot 17 =$	1 point	$S_{17} = \frac{2 \cdot 12 + 16 \cdot 3}{2} \cdot 17 =$
= 612 saplings.	1 point	
Total:	**5 points**	

Note: Award full marks if the candidate gives the correct answer by listing the first 17 terms of the sequence.

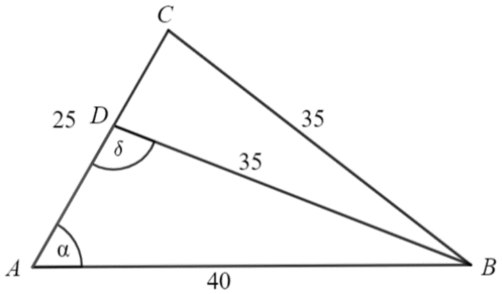
II. B

16. a)		
The central angle that belongs to absolute values is $360° \cdot \frac{5}{24} = 75°$.	1 point	*Each function has a central angle of* $\frac{360°}{24} = 15°$, *so the central angle that belongs to the absolute value functions is* $5 \cdot 15° = 75°$,
The central angle that belongs to the quadratic functions is $360° \cdot \frac{6}{24} = 90°$.	1 point	*the one that belongs to the quadratic functions is* $6 \cdot 15° = 90°$.
There are $24 \cdot \frac{135°}{360°} = 9$ linear functions.	1 point	$\frac{135°}{15°} = 9$
The rest, 24 – (5 + 9 + 6) = 4 functions are radical.	1 point	
The central angle that belongs to radical functions is (360° – 75° – 135° – 90° =) 60°.	1 point	$4 \cdot 15° = 60°$
Correctly labelled pie chart, such as: 	2 points	
Total:	**7 points**	

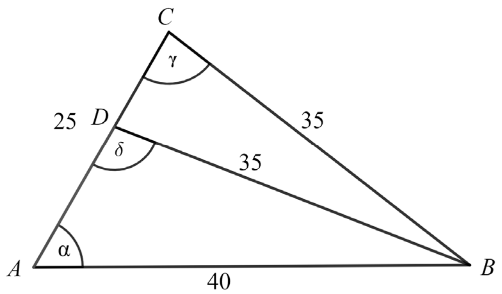
16. b)		
The graph of function f: 	2 points	*Zeros are given by the solutions of the equation* $(x-3)^2-4=0$. *Rearranged:* $x^2-6x+5=0$.
Zeros: $x_1 = 1$ and $x_2 = 5$.	1 point	
Function f is (strictly monotone) decreasing, where $x \leq 3$,	1 point	*Accept* $x < 3$.
and (strictly monotone) increasing, where $x \geq 3$.	1 point	*Accept* $x > 3$.
The function has a minimum	1 point	
at (x =) 3,	1 point	
the minimum value is ($f(3)$ =) –4.	1 point	
The range of the function is [–4; ∞ [.	2 points	*Accept the correct answer in any form.*
Total:	**10 points**	

17. a)		
Apply the Cosine Rule for side BC in triangle ABC: $35^2 = 40^2 + 25^2 - 2\cdot 40 \cdot 25 \cdot \cos\alpha$.	1 point	
Hence $\cos\alpha = 0.5$,	2 points	
i.e. $\alpha = 60°$.	1 point	
Total:	**4 points**	

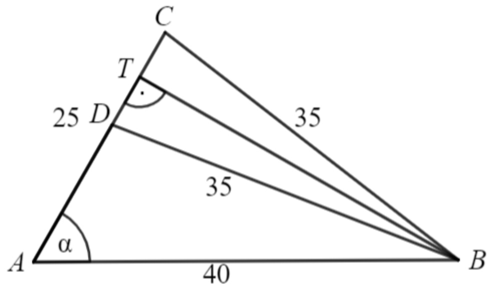
Note: Award 3 points if the candidate makes a true statement by substituting 60° into the Cosine Rule. Award a further 1 point if the candidate also refers to there being no other possible angles.

17. b) **Solution 1**		
 Sine Rule in triangle *ABD*: $\frac{\sin\delta}{\sin 60°}=\frac{40}{35}$.	1 point*	
$\sin\delta \approx 0.9897$.	1 point*	
$\delta \approx 81.8°$ is not acceptable (as triangle *ABD* would not be obtuse then).	1 point*	*Award this point if the correct reasoning is reflected only by the solution.*
$\delta = (180° - 81.8° =)\ 98.2°$ is correct.	1 point*	
The angle at vertex *B* in triangle *ABD* is $(180° - 60° - 98.2° =)\ 21.8°$.	1 point	
The area of the triangle is: $A_{ABD}=\frac{40\cdot 35\cdot \sin 21.8°}{2}\approx 260\ \text{cm}^2$.	2 points	
Total:	**7 points**	

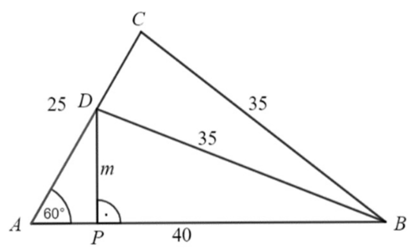
*The 4 points marked * may also be given for the following reasoning:*

 Let γ be the interior angle at vertex *C* of triangle *ABC*. Apply the Cosine Rule: $40^2=35^2+25^2-2\cdot 35\cdot 25\cdot\cos\gamma$.	1 point	
$\cos\gamma \approx 0.1429$	1 point	
$\gamma \approx 81.8°$	1 point	
As triangle *BCD* is isosceles, $\delta = 180° - \gamma = 98.2°$.	1 point	

17. b) **Solution 2**		
Apply the Cosine Rule in triangle *ABD*: $35^2 = 40^2 + AD^2 - 2 \cdot 40 \cdot AD \cdot \cos 60°$.	1 point	
$AD^2 - 40 \cdot AD + 375 = 0$.	2 points	
AD = 25 is incorrect (as *AD* < *AC*).	1 point	
AD = 15 (cm) is correct.	1 point	
The area of the triangle: $A_{ABD} = \frac{15 \cdot 40 \cdot \sin 60°}{2} \approx 260$ cm^2.	2 points	
Total:	**7 points**	

17. b) **Solution 3**		
 Draw the height of triangle *ABD* from vertex *B*. (As triangle *ABD* is obtuse, the base *T* of this height will be outside this triangle.)	1 point	
Triangle *ABT* is half of a regular triangle, therefore *AT* = 20 (cm).	1 point	
CT = *AC* – *AT* = 5 (cm).	1 point	$CT = \sqrt{35^2 - (20 \cdot \sqrt{3})^2}$
(In the isosceles triangle *BCD* point *T* bisects segment *CD*, so) *DT* = 5 (cm) and *AD* = 15 (cm).	1 point	
BT = $20 \cdot \sqrt{3}$ (cm)	1 point	
The area of triangle *ADB* is $A_{ABD} = \frac{AD \cdot BT}{2} = 150 \cdot \sqrt{3} \approx 260$ cm^2.	2 points	
Total:	**7 points**	

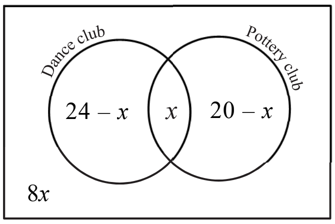
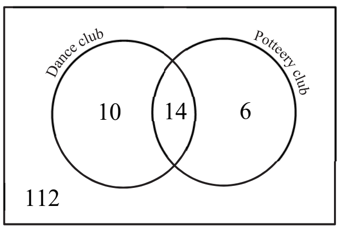
Note: Let m be the height that belongs to side AB in triangle ABD. In this case the length of segment AP is $\frac{m}{\sqrt{3}}$*, the length of segment PB is* $40 - \frac{m}{\sqrt{3}}$ *(1 point). Apply the Pythagorean Theorem in triangle PBD (1 point), hence obtaining* $m = \frac{15 \cdot \sqrt{3}}{2}$ *≈ 13 (cm) (3 points). From the length of side AB and the height that belongs to it the area of triangle ABD can be obtained (2 points).*



17. c) **Solution 1**		
Possible routes are: *BCDBAD*; *BCDABD*; *BDCBAD*; *BDABCD*; *BADBCD*; *BADCBD*.	3 points	*Award 2 points for 4 or 5 correct routes, 1 point for 3 correct routes, 0 points for less than 3 correct routes. Deduce a maximum of 1 point if the candidate also lists any incorrect route(s).*
There are a total of 6 different possible routes.	1 point	
Total:	**4 points**	

17. c) **Solution 2**		
There are 3 possible ways to travel from point B to point D (two of which travels along 2 edges, and the third on only 1 edge).	1 point	
There are 2 ways left then to return from D to B,	1 point	
and only one choice left to travel to D again.	1 point	
There are a total of $3 \cdot 2 \cdot 1 = 6$ possible routes.	1 point	
Total:	**4 points**	

17. d)		
(1) true (2) false (3) true	2 points	*Award 2 points for two or three correct answers, 1 point for one correct answer.*
Total:	**2 points**	

18. a)		
Let x be the number of students attending both clubs. In this case, the number of students attending neither club is $8x$.	1 point	
The number of students attending the dance club only is $24 - x$, the number of students attending the pottery club only is $20 - x$.	1 point	
According to the problem: $24 - x + 20 - x + x + 8x = 142$.	1 point	$24 + 20 - x + 8x = 142$
$x = 14$	1 point	
There are 10 students attending the dance club only,	1 point	
and 6 students attending the pottery club only.	1 point	
Check against the text: $10 + 6 + 14 + 8 \cdot 14 = 142$.	1 point	
Total:	**7 points**	

18. b) Solution 1		
The first student may sit to any of the 16 spaces. The next student must not sit next to the first, so only 14 spaces are still available, and so on.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The total number of possible seating arrangements is therefore $16 \cdot 14 \cdot 12 \cdot \ldots \cdot 2 =$	2 points	
$= 10\ 321\ 920$.	1 point	
Total:	**4 points**	

18. b) Solution 2		
There are 8! (= 40 320) different possible ways to select a desk for each of the eight students.	1 point	
At each desk, there are 2 seats to choose from. For 8 students, this makes 2^8 (= 256) possibilities.	2 points	
The total number of possible seating arrangements is therefore $2^8 \cdot 8! = 256 \cdot 40320 = 10\ 321\ 920$.	1 point	
Total:	**4 points**	

18. c)		
There are $\binom{14}{3}$ (= 364) different possible ways to select 3 students out of 14 (disregarding order). This is the total number of cases.	1 point	*With regards to the order:* $14 \cdot 13 \cdot 12$
There are $\binom{6}{2}$ (= 15) different ways to select 2 out of the 6 students attending the pottery club.	1 point	$6 \cdot 5$
There are 8 different ways to select the remaining 1 student from the other 8.	1 point	
The number of favourable cases is $\binom{6}{2} \cdot 8$.	1 point	$6 \cdot 5 \cdot 8 \cdot 3$
The probability: $\dfrac{\binom{6}{2} \cdot 8}{\binom{14}{3}} = \dfrac{120}{364} \approx 0.33$.	2 points	$\dfrac{6 \cdot 5 \cdot 8 \cdot 3}{14 \cdot 13 \cdot 12} = \dfrac{720}{2184}$
Total:	**6 points**	